Appendix 3

Suppose that the responses to a given biomarker follow a gamma distribution such that cases are $Gamma(\alpha_x, \beta_x)$ and controls are $Gamma(\alpha_y, \beta_y)$, where

$$Gamma(x;\alpha,\beta) = \frac{e^{-x/\beta} x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)},$$

with $\Gamma(n)$ being the gamma function and $\mu_x > \mu_y$; where $\mu_x = \alpha_x \times \beta_x$ and $\mu_Y = \alpha_Y \times \beta_Y$.

Based on these distributional assumptions, sensitivity ($q(c)$) and specificity ($p(c)$) become

$$q(c) = P(X \geq c) = \frac{1}{\Gamma(\alpha_X)(\beta_X)^{\alpha_X}} \int_c^{\infty} x^{\alpha_X - 1} e^{-x/\beta_X} dx, \qquad (6)$$

and

$$p(c) = P(Y < c) = \frac{1}{\Gamma(\alpha_Y)(\beta_Y)^{\alpha_Y}} \int_0^{c} y^{\alpha_Y - 1} e^{-y/\beta_Y} dy. \qquad (7)$$