eAppendix 1A: Analytic Comparison of Estimators

It is straightforward to compare $\hat{\beta}_{CME}$ and $\hat{\beta}_{SA}$ because their numerators are the same and the denominator of $\hat{\beta}_{CME}$ differs from the denominator of $\hat{\beta}_{SA}$ only by the term $-\frac{1}{2}\sum_{j=1}^{2}\sum_{i=1}^{n}(X_{ij}-\bar{X}_{i\cdot})^2$. More formally,

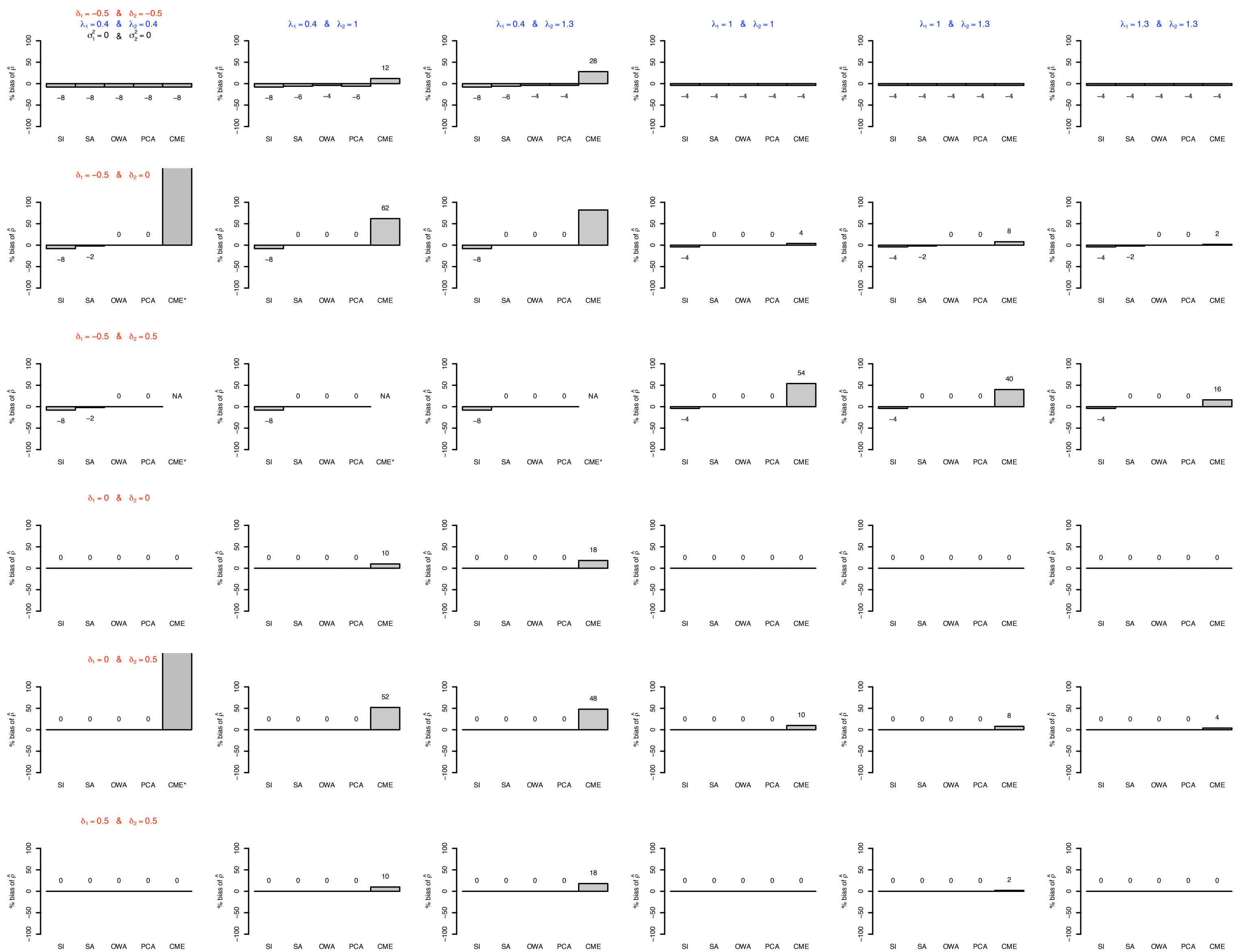
$$\hat{\beta}_{CME} = \left[\hat{\beta}_{SA}^{-1} - \frac{\frac{1}{2}\sum_{j=1}^{2}\sum_{i=1}^{n}(X_{ij}-\bar{X}_{i\cdot})^2}{\sum_{i=1}^{n}(\bar{X}_{i\cdot}-\bar{X}_{\cdot\cdot})(Y_i-\bar{Y})}\right]^{-1}.$$

Thus, we see that $\hat{\beta}_{CME} = \hat{\beta}_{SA}$ only in the limiting case where $\delta_1 = \delta_2 = 0$, $\sigma_1^2 = \sigma_2^2 = 0$ and $\lambda_1 = \lambda_2 = 1$. Further, since the term $-\frac{1}{2}\sum_{j=1}^{2}\sum_{i=1}^{n}(X_{ij}-\bar{X}_{i\cdot})^2$ is never positive, we can see that $\hat{\beta}_{SA}$ will always be (weakly) smaller in magnitude (i.e. closer to 0) than $\hat{\beta}_{CME}$.

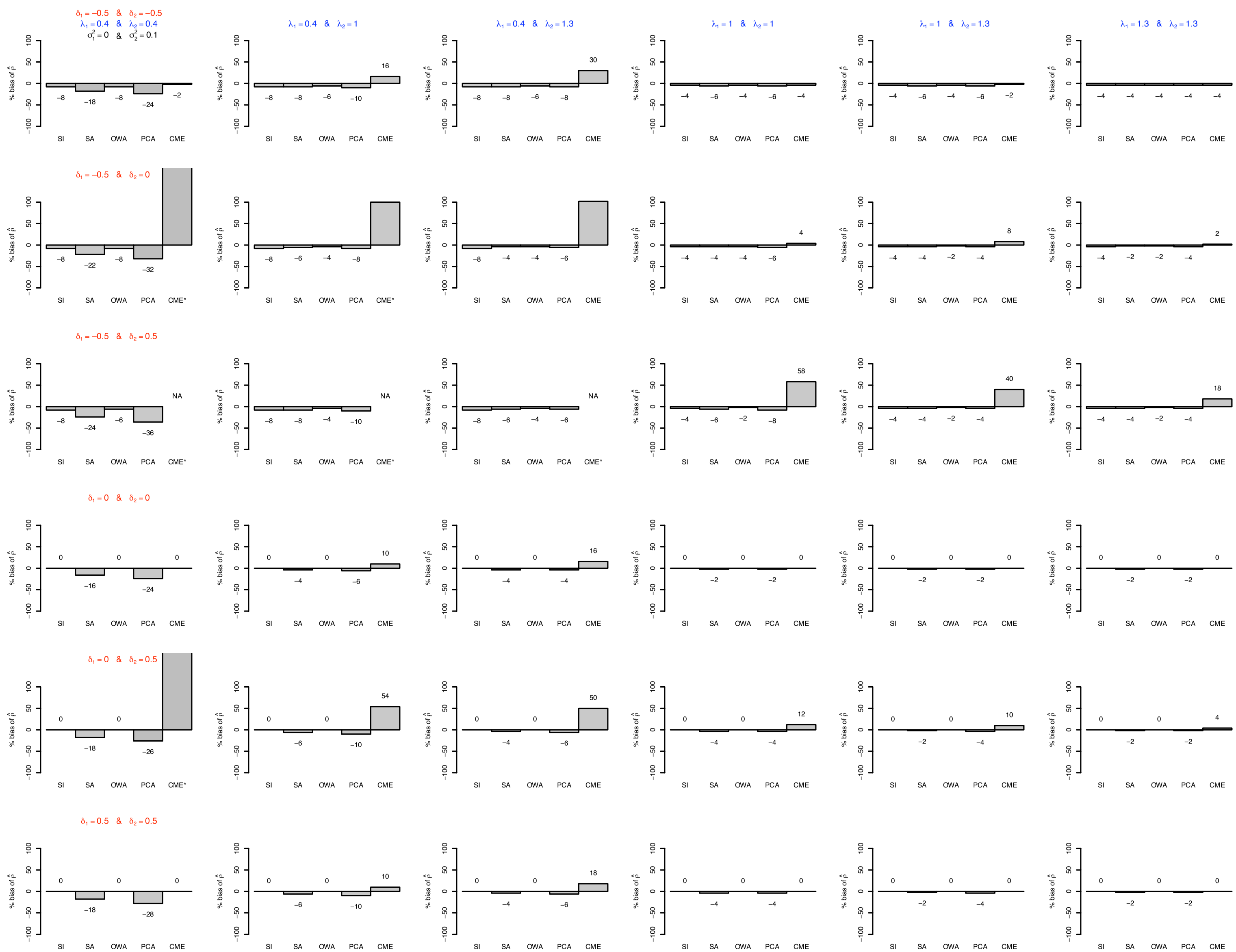
eAppendix 1B: Simulation Experiments

For every combination of the above parameter values, we generated 1000 datasets of size $n = 250$. To create each dataset, we first generated the unobserved true predictor (the T_is) from a normal distribution with mean 0 and variance 1, with sub-zero values rounded up to 0, which yielded a distribution similar to the one in the VPA and BMI example. Next, we generated the observed measures of the true predictor (the X_{i1}s and X_{i2}s) by generating ϵ_{i1} and ϵ_{i2} from normal distributions with mean 0 and variances σ_1^2 and σ_2^2, respectively, and then using Equation (1) with the relevant values of δ_1, δ_2, λ_1, and λ_2 to calculate X_{i1} and X_{i2} from T_i, ϵ_{i1}, and ϵ_{i2}. Last, we generated the outcomes (the Y_is) by generating ε_i from a $N(0, \sigma_\varepsilon^2 = 1)$ distribution and then using Equation (1) with $\alpha = 1$ and $\beta = 1$ to combine T_i and ε_i. For each dataset, the SI, SA, OWA, PCA, and CME methods were used to estimate β and ρ by applying the formulas in Section 2 to Y_i and X_{i1} and/or X_{i2}. The OWA method was implemented by estimating α and β (or ρ) for each value of w_1 in a grid of values ranging from 0 to 1 in 0.05 increments, and then selecting the estimate that maximized the expression in Equation (4). For each method, the differences or the squared differences between the resulting estimates and the true value of β (= 1) or ρ (= 0.5) were then averaged over the 1000 datasets to calculate the average bias and the average MSE, respectively.

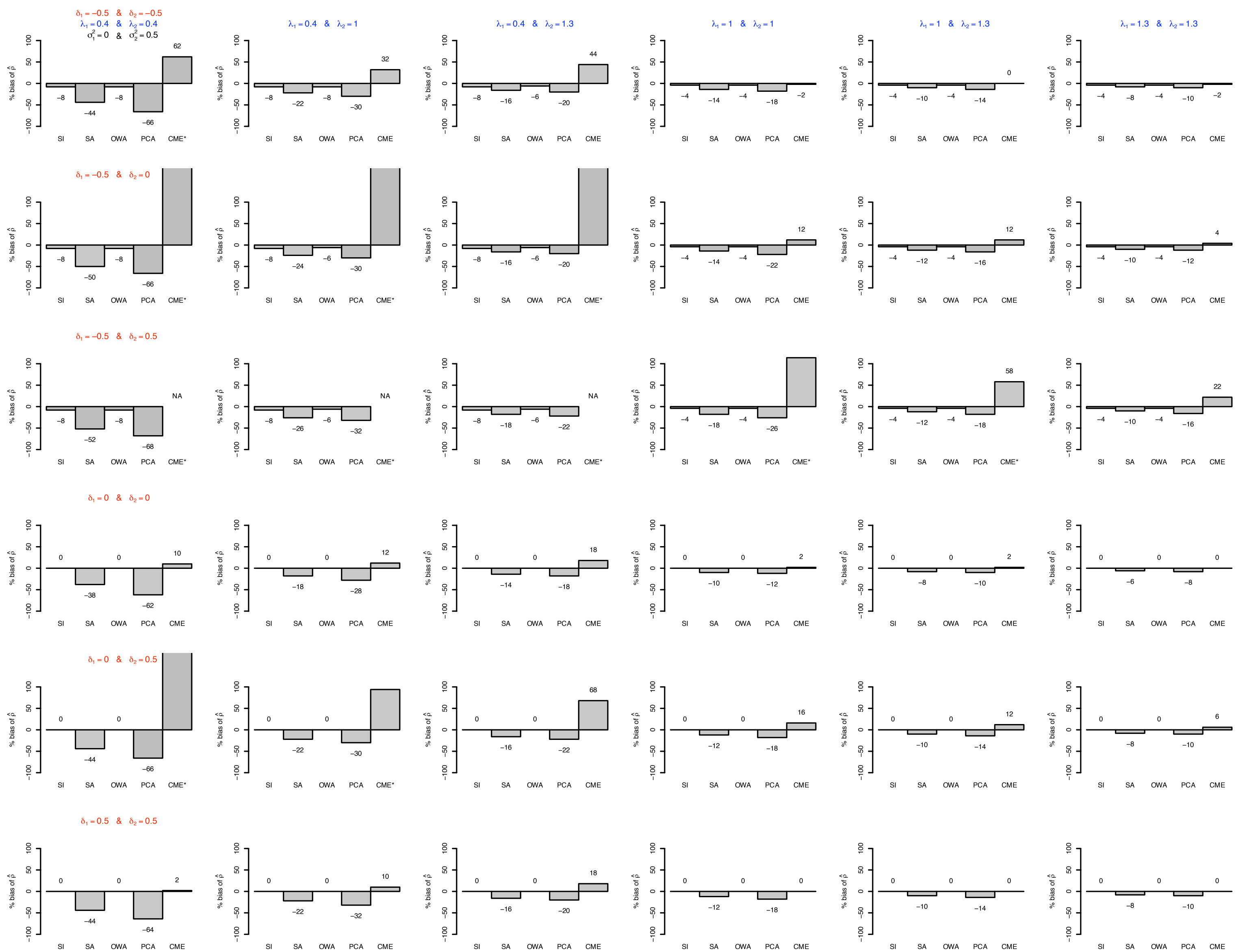
eFigure 1: Average Percent Bias of $\hat{\rho}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method. *Each figure presents the average percent bias of* $\hat{\rho}$*, by method, for* $\sigma_1^2 = 0.1$ *and* $\sigma_2^2 = 0.1$ *and a particular combination of* δ_1*,* δ_2*,* λ_1*, and* λ_2*. The rows in which the figures are arranged correspond to various combinations of* δ_1 *and* δ_2*, and the columns in which the figures are arranged correspond to various combinations of* λ_1 *and* λ_2*. An asterisk following the name of a method refers to scenarios where the estimation method failed in at least some of the* 1000 *iterations, and 'NA' refers to scenarios where the estimation method failed in all of the* 1000 *iterations.*



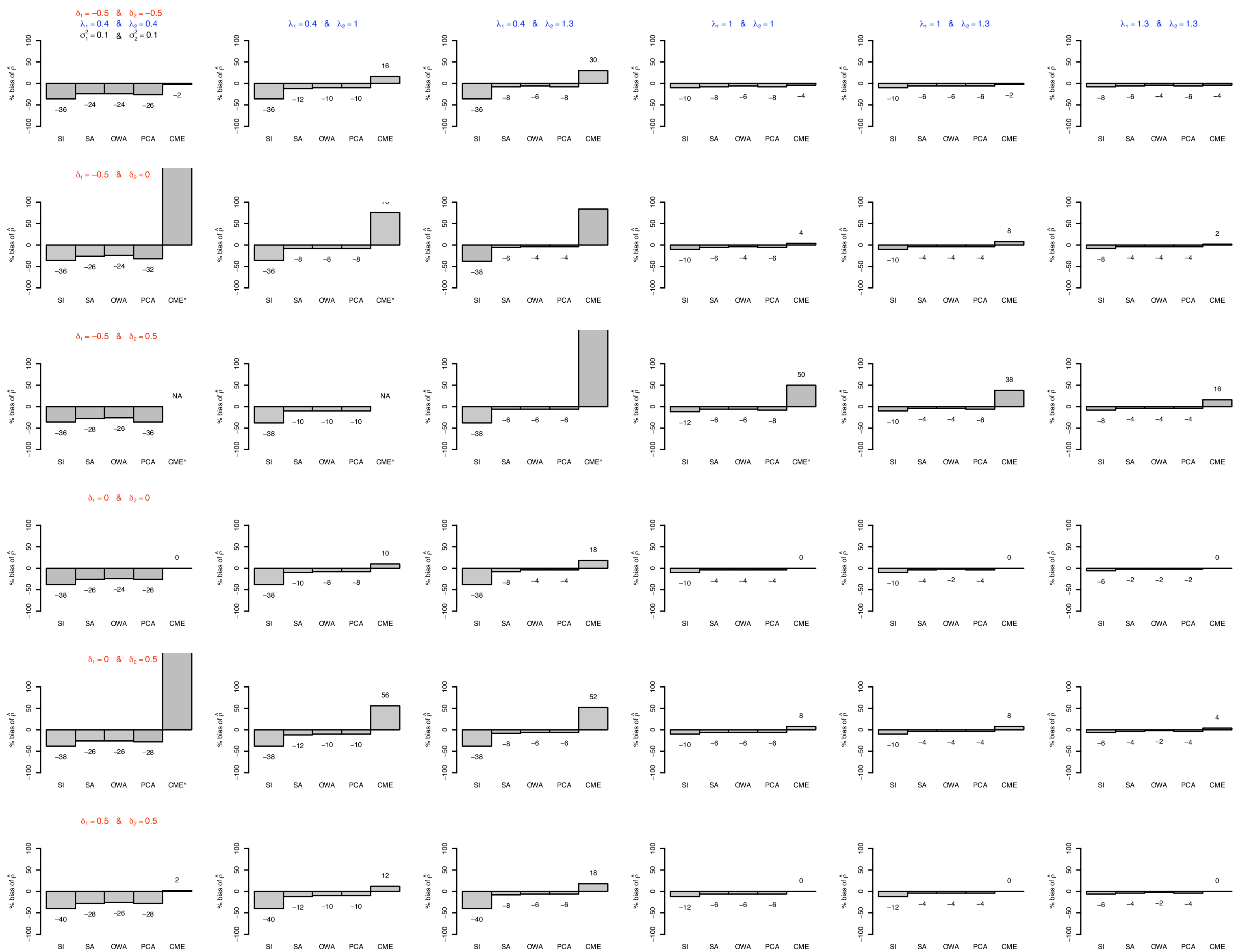
eFigure 2: Average Percent Bias of $\hat{\rho}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



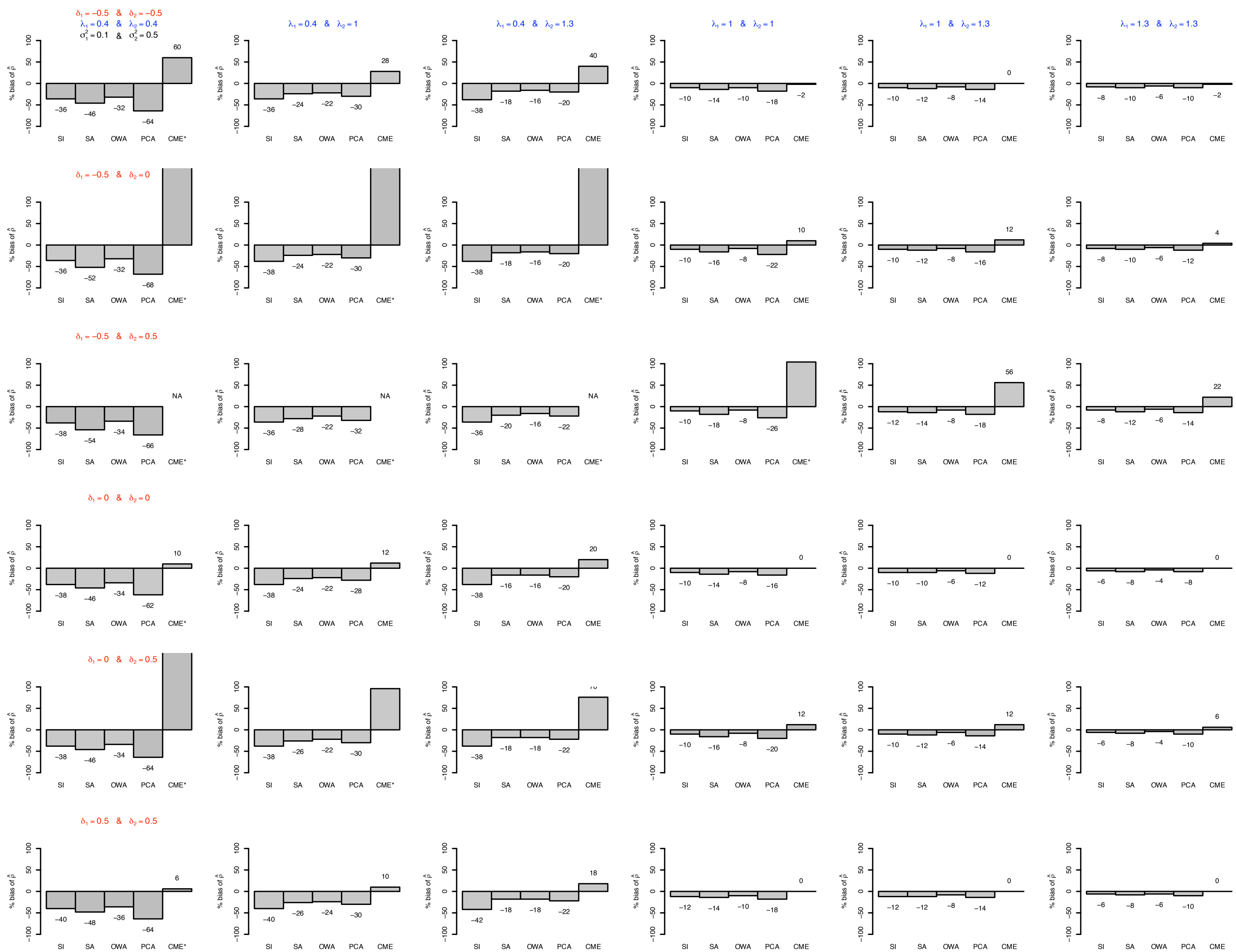
eFigure 3: Average Percent Bias of $\hat{\rho}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



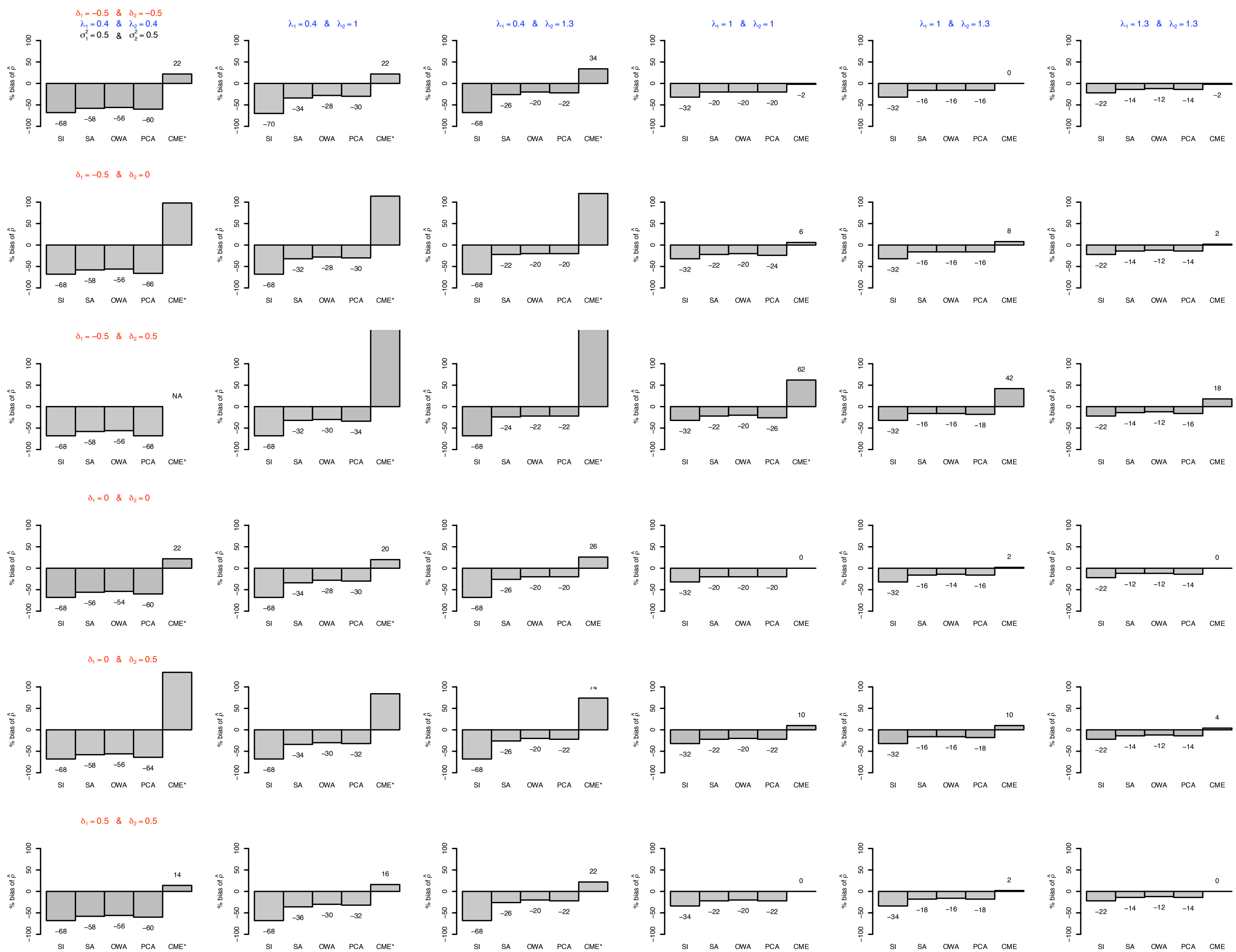
eFigure 4: Average Percent Bias of $\hat{\rho}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



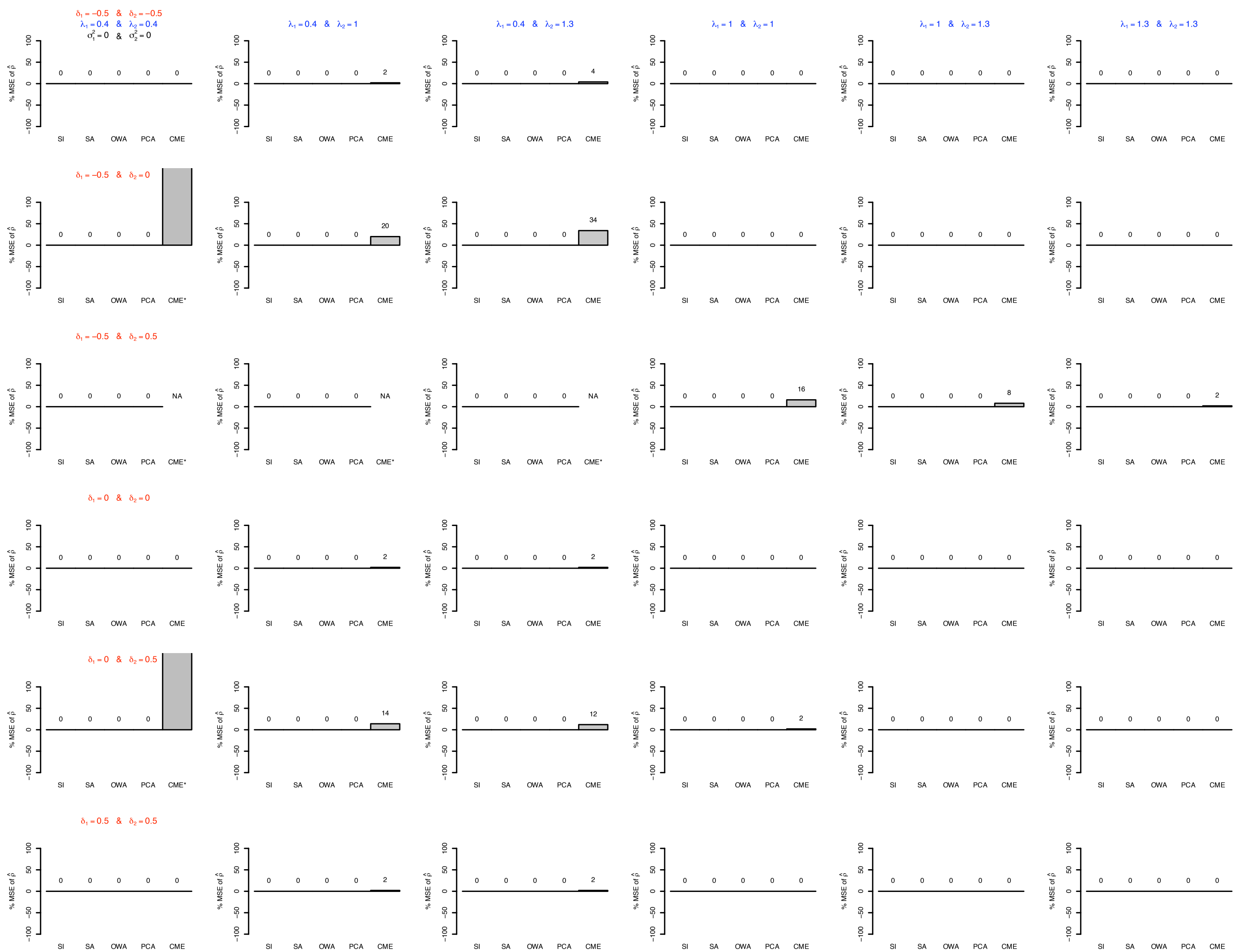
eFigure 5: Average Percent Bias of $\hat{\rho}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



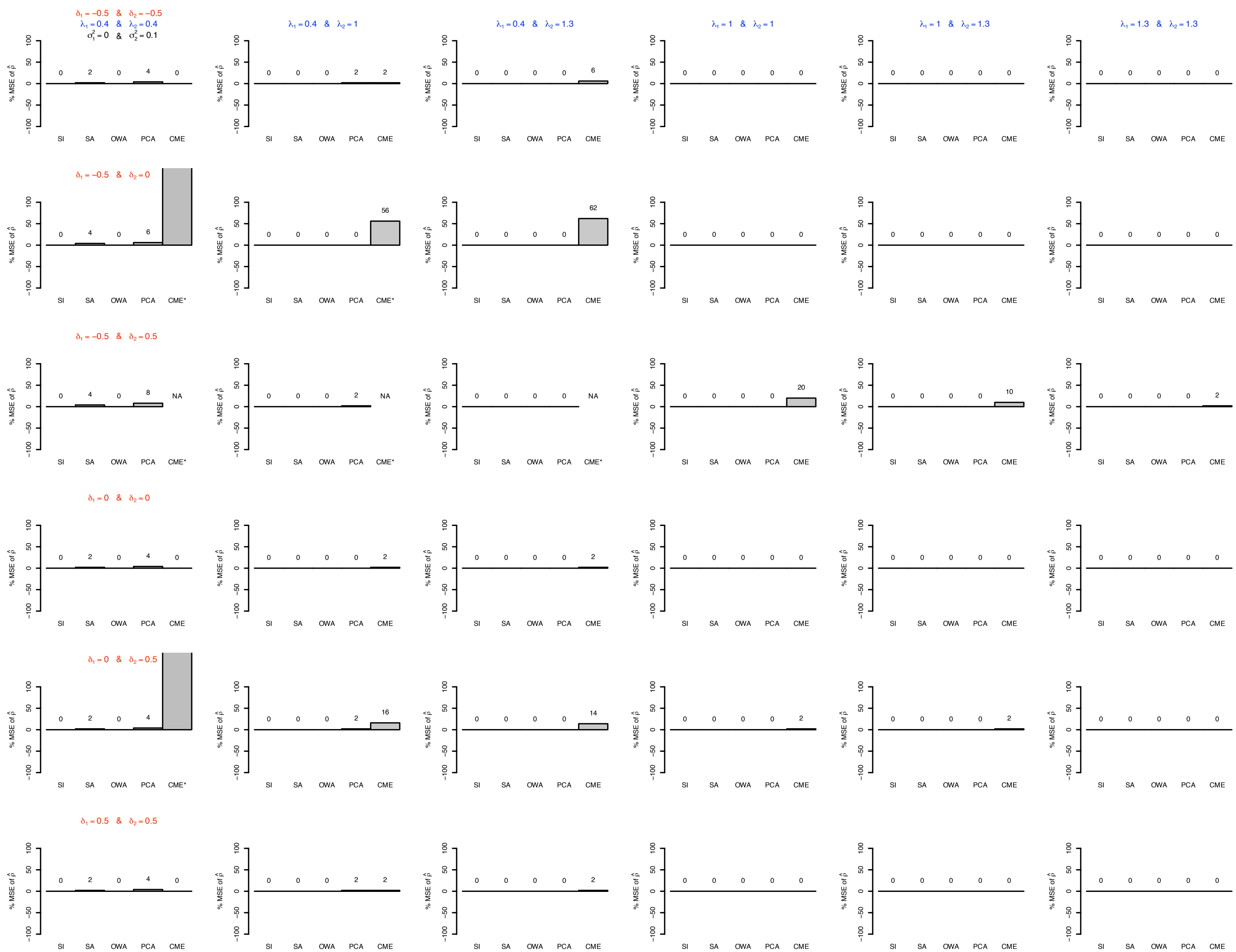
eFigure 6: Average Percent Bias of $\hat{\rho}$ for $\sigma_1^2 = 0.5$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



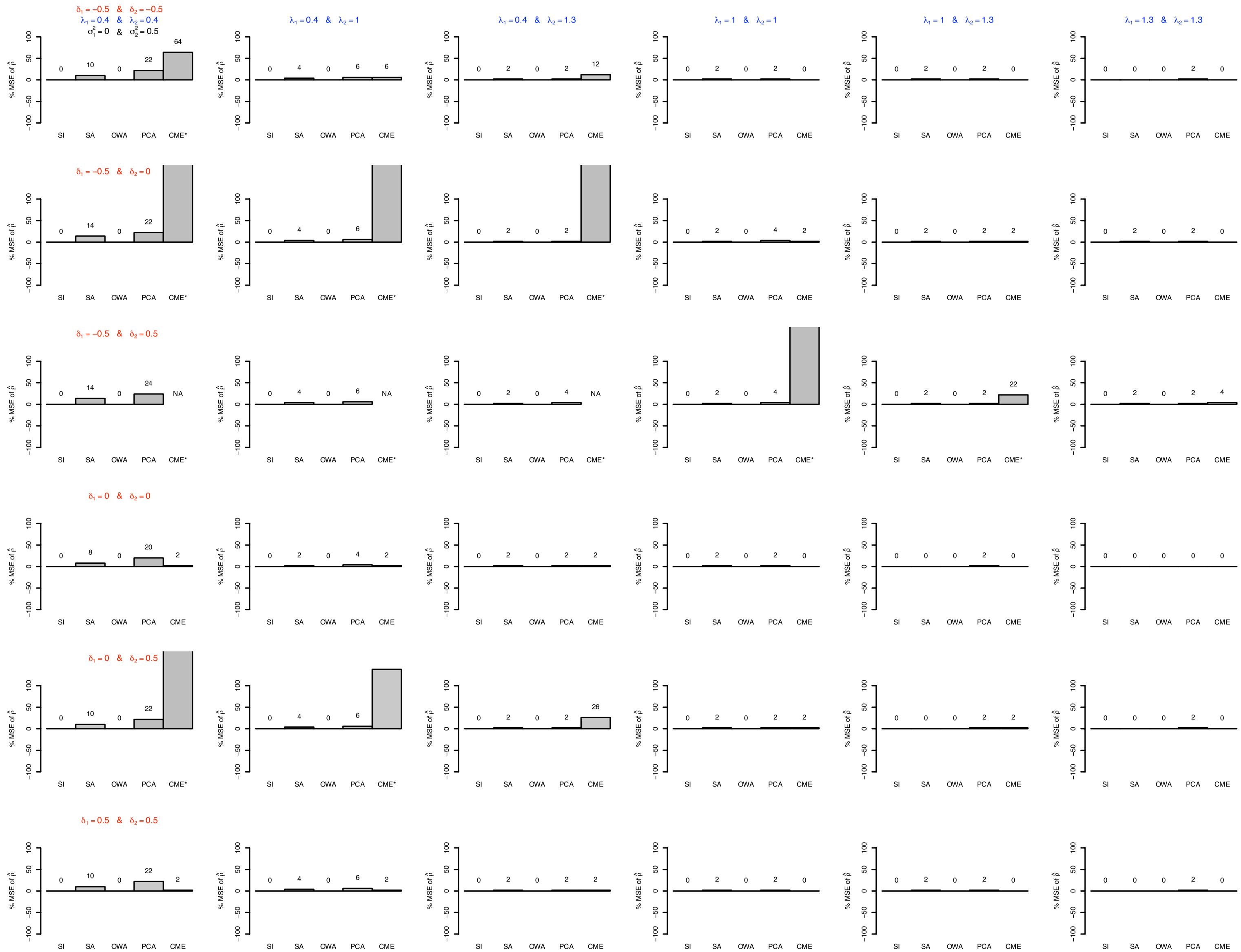
eFigure 7: Average Percent Mean Squared Error (MSE) of $\hat{\rho}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method. *Each figure presents the average percent MSE of $\hat{\rho}$, by method, for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0$ and a particular combination of δ_1, δ_2, λ_1, and λ_2. The rows in which the figures are arranged correspond to various combinations of δ_1 and δ_2, and the columns in which the figures are arranged correspond to various combinations of λ_1 and λ_2. An asterisk following the name of a method refers to scenarios where the estimation method failed in at least some of the* 1000 *iterations, and 'NA' refers to scenarios where the estimation method failed in all of the* 1000 *iterations.*



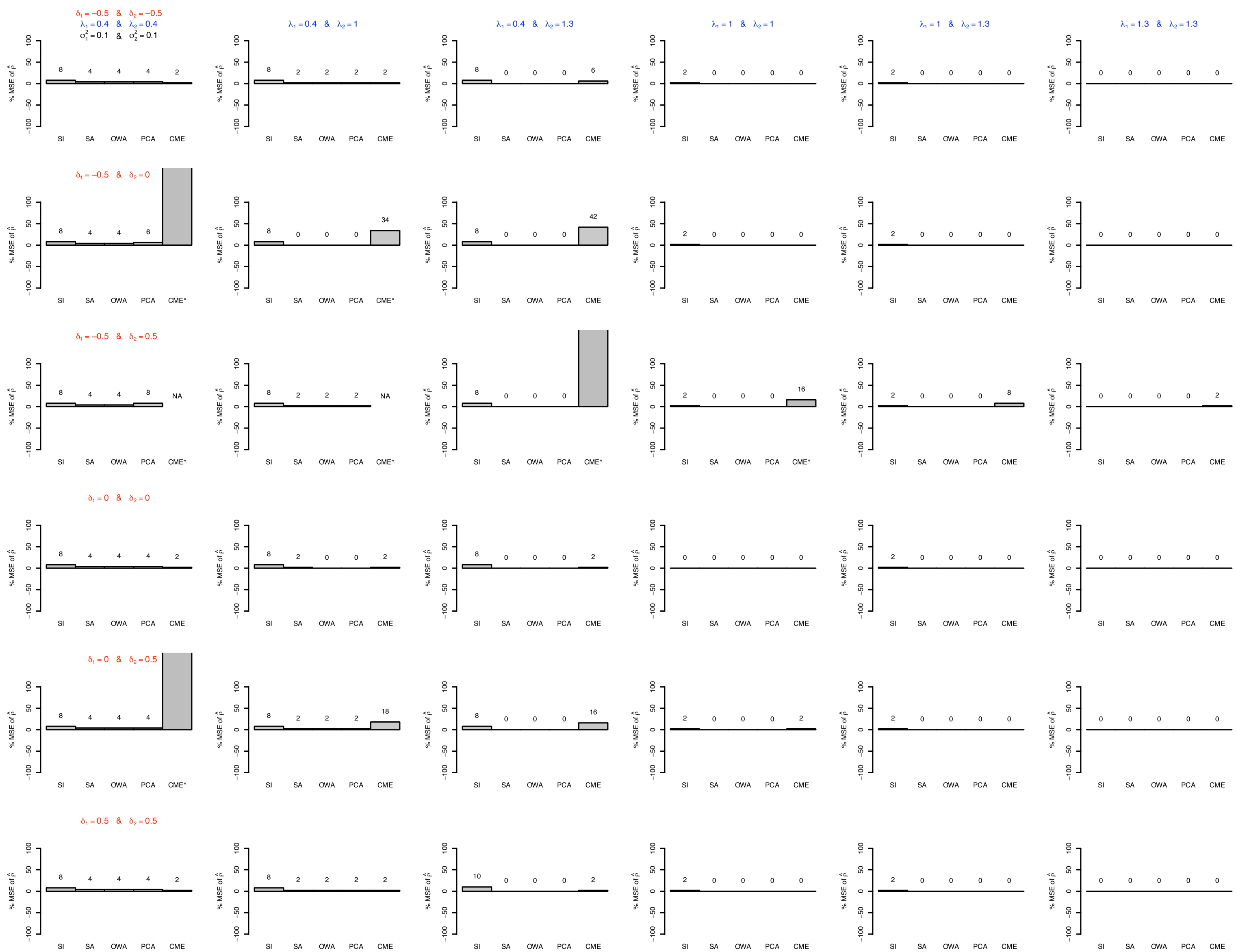
eFigure 8: Average Percent Mean Squared Error (MSE) of $\hat{\rho}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



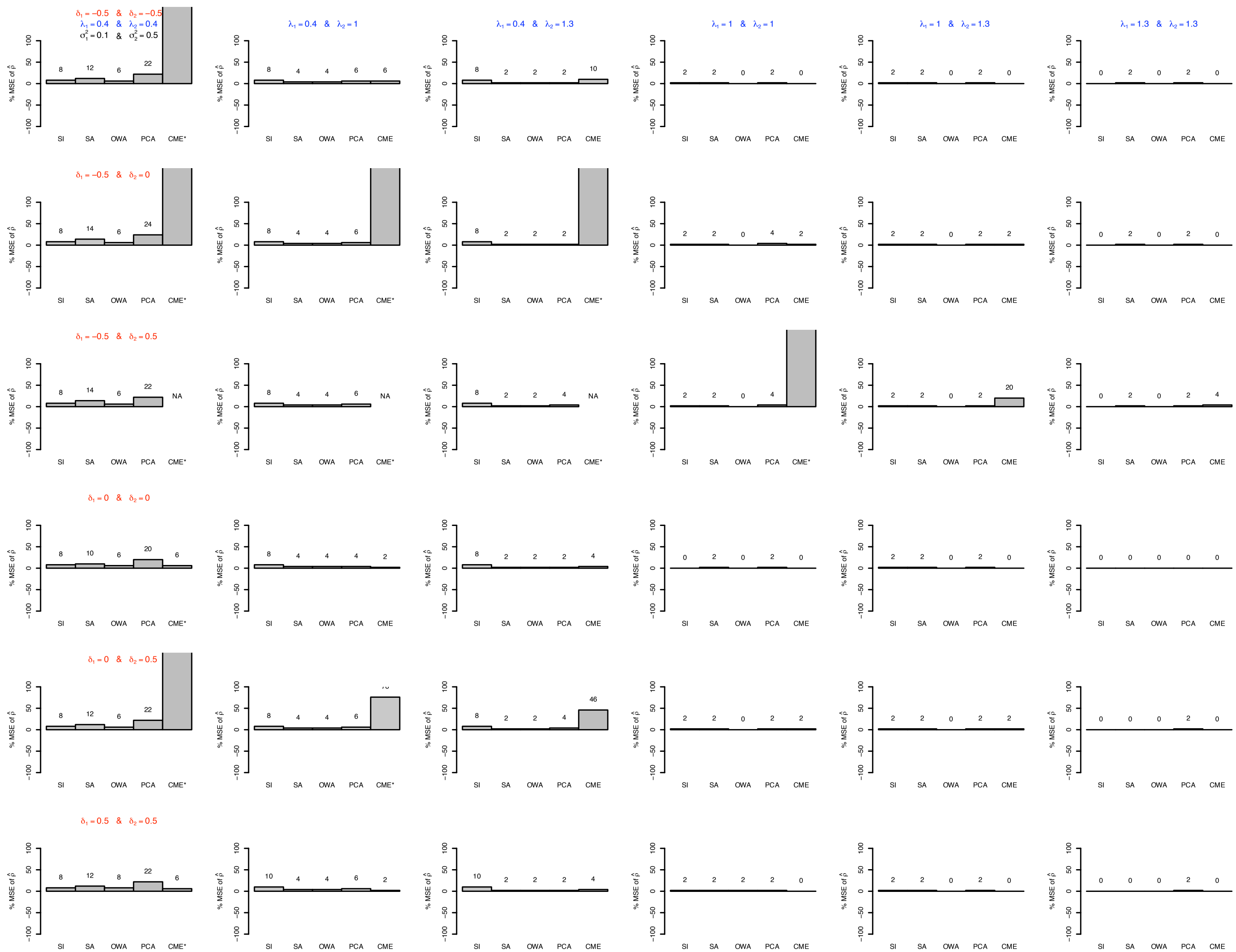
eFigure 9: Average Percent Mean Squared Error (MSE) of $\hat{\rho}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



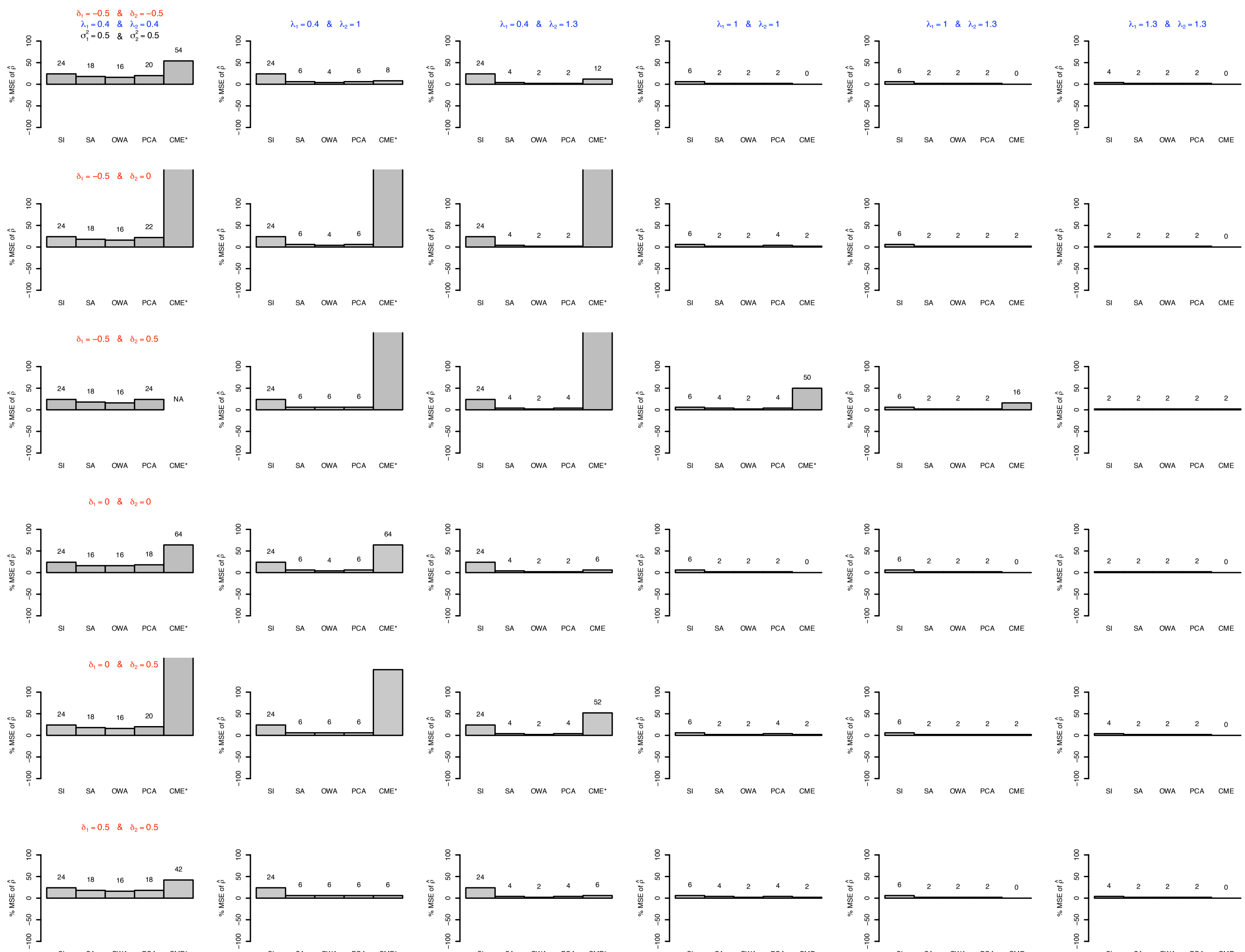
eFigure 10: Average Percent Mean Squared Error (MSE) of $\hat{\rho}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



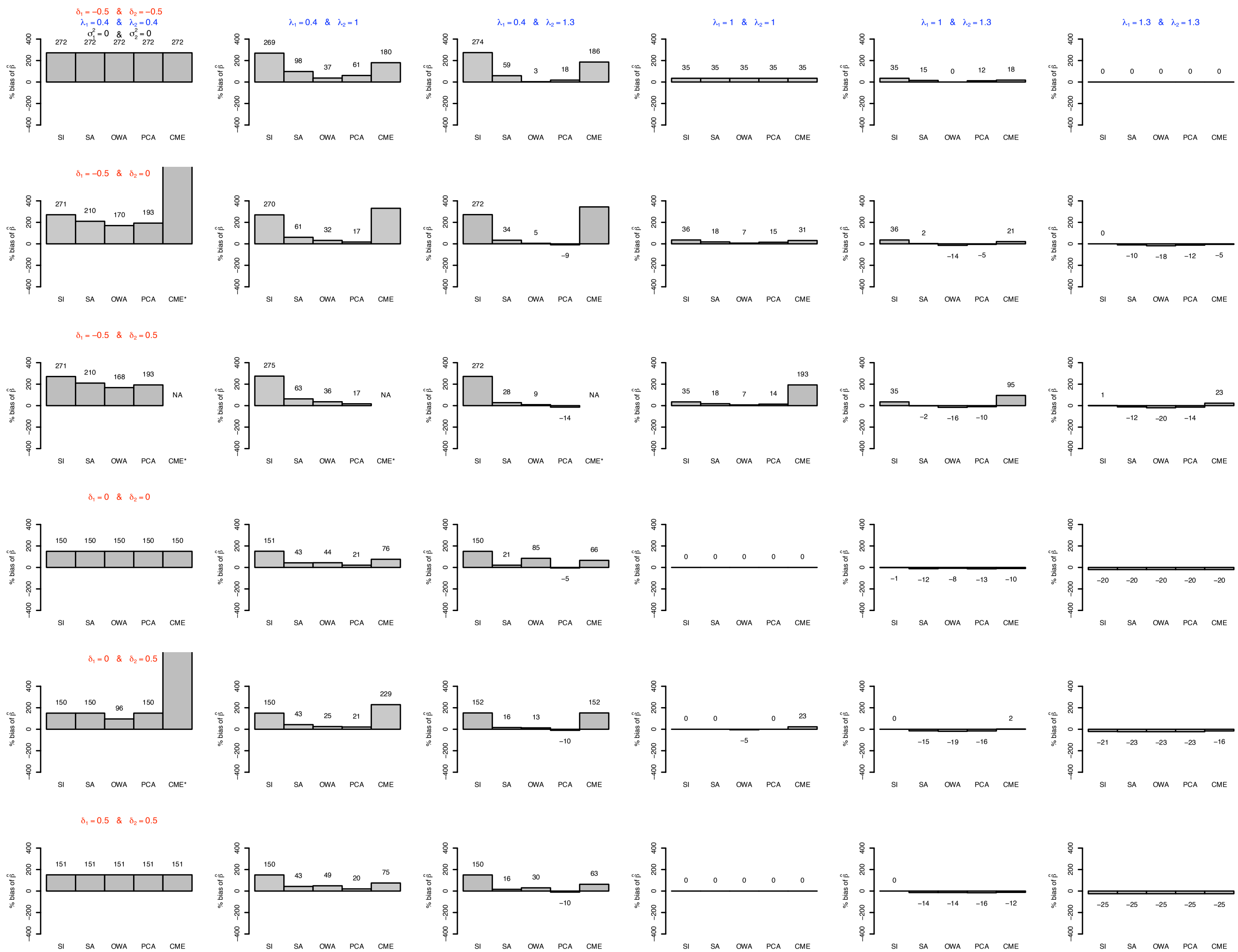
eFigure 11: Average Percent Mean Squared Error (MSE) of $\hat{\rho}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



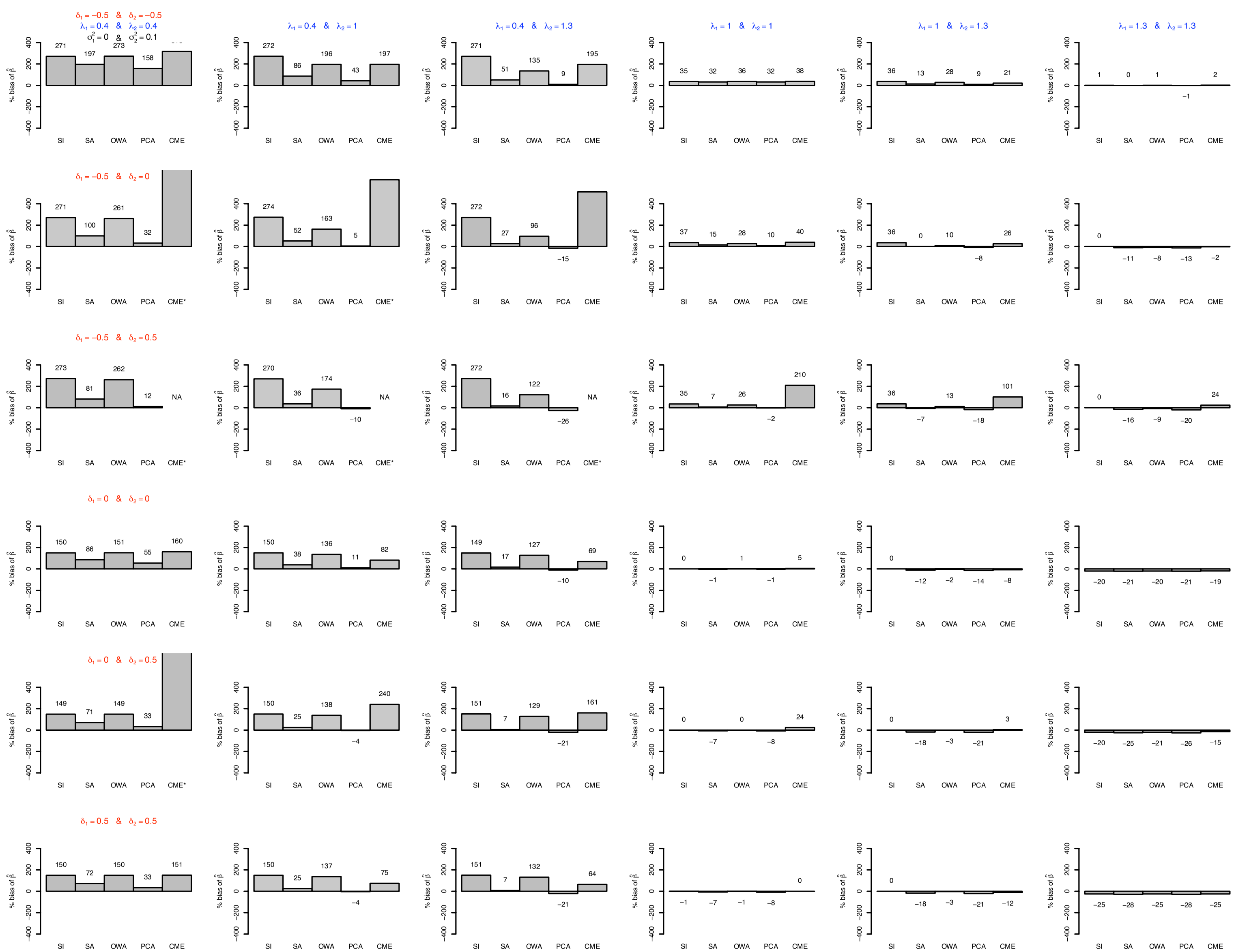
eFigure 12: Average Percent Mean Squared Error (MSE) of $\hat{\rho}$ for $\sigma_1^2 = 0.5$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



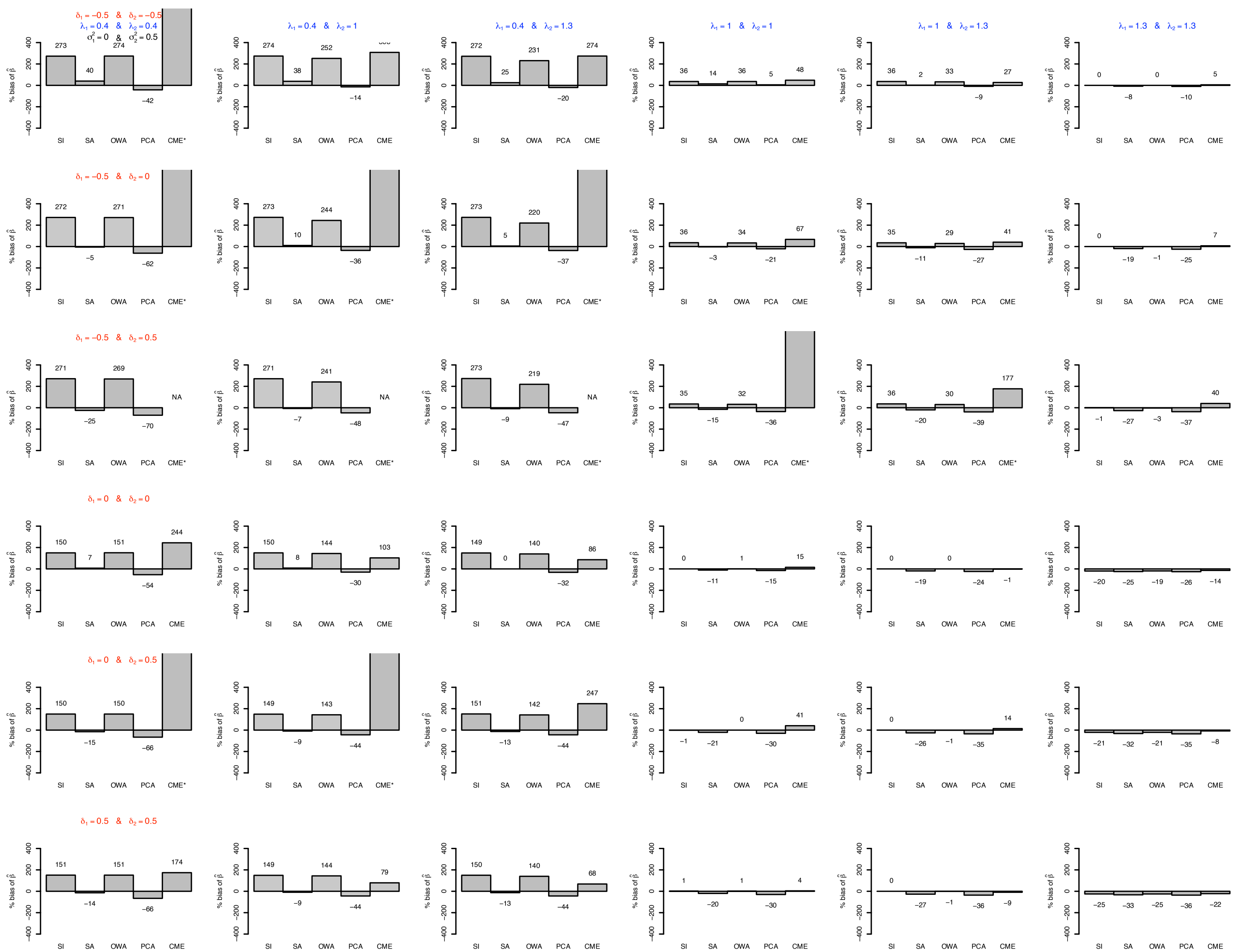
eFigure 13: Average Percent Bias of $\hat{\beta}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



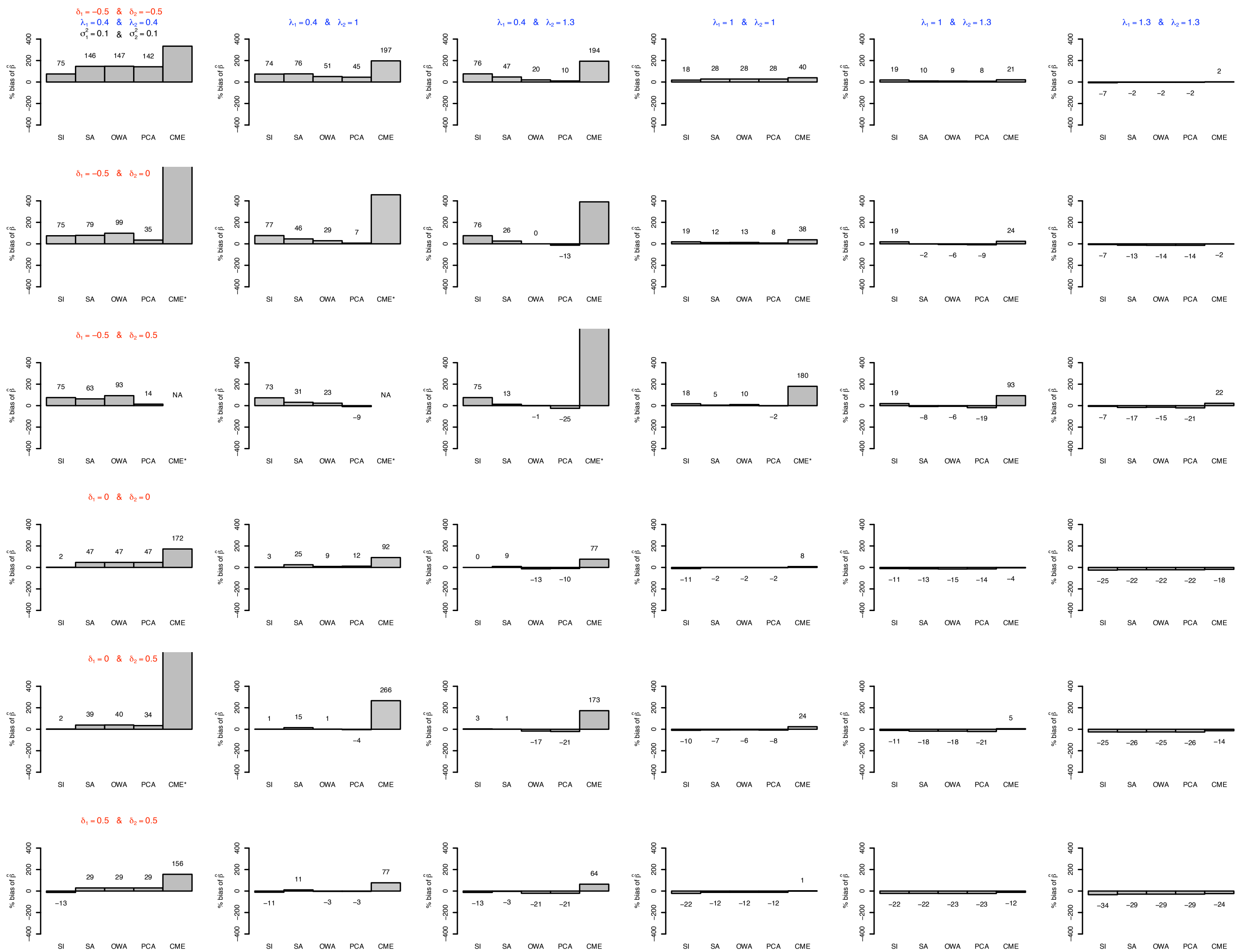
eFigure 14: Average Percent Bias of $\hat{\beta}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



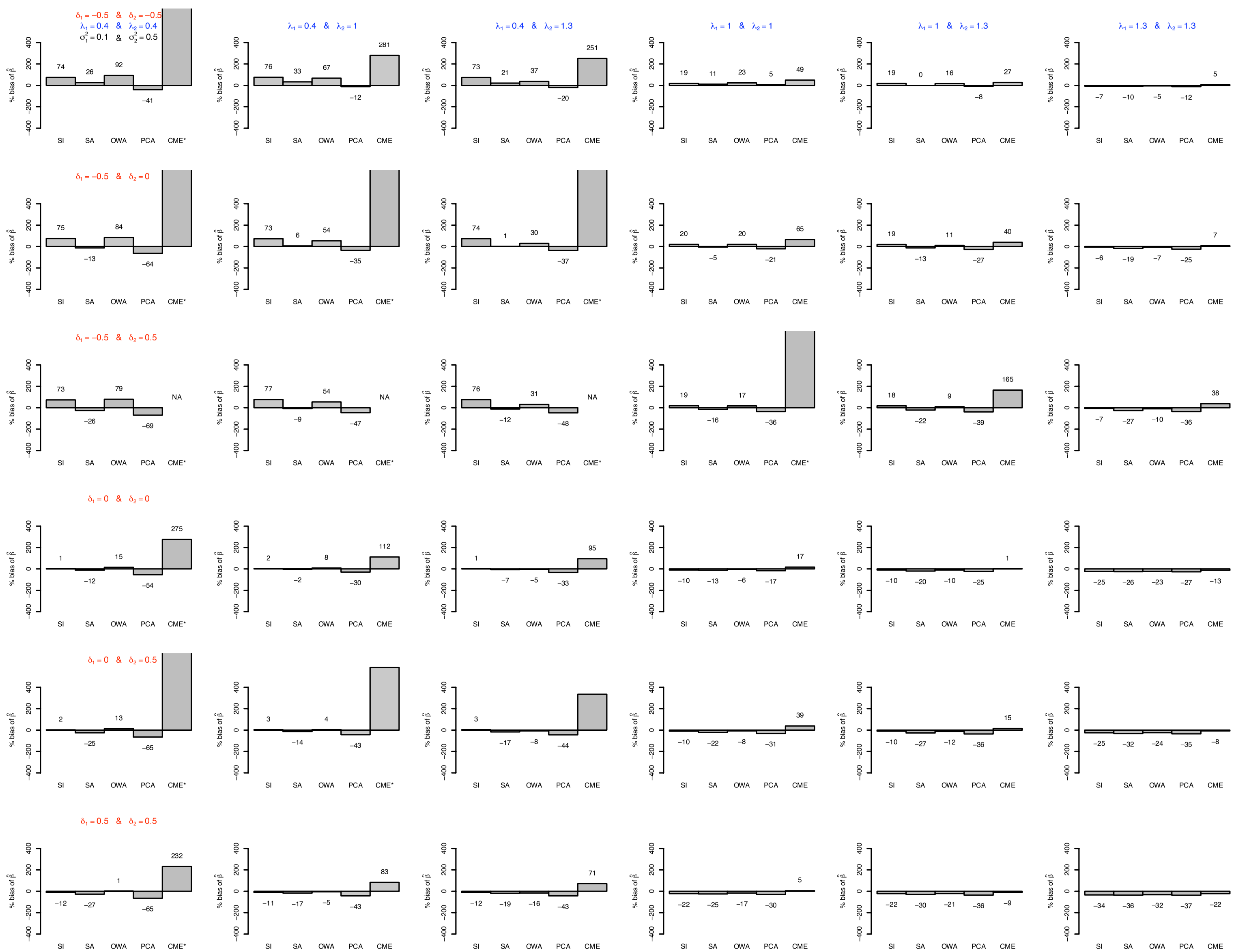
eFigure 15: Average Percent Bias of $\hat{\beta}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



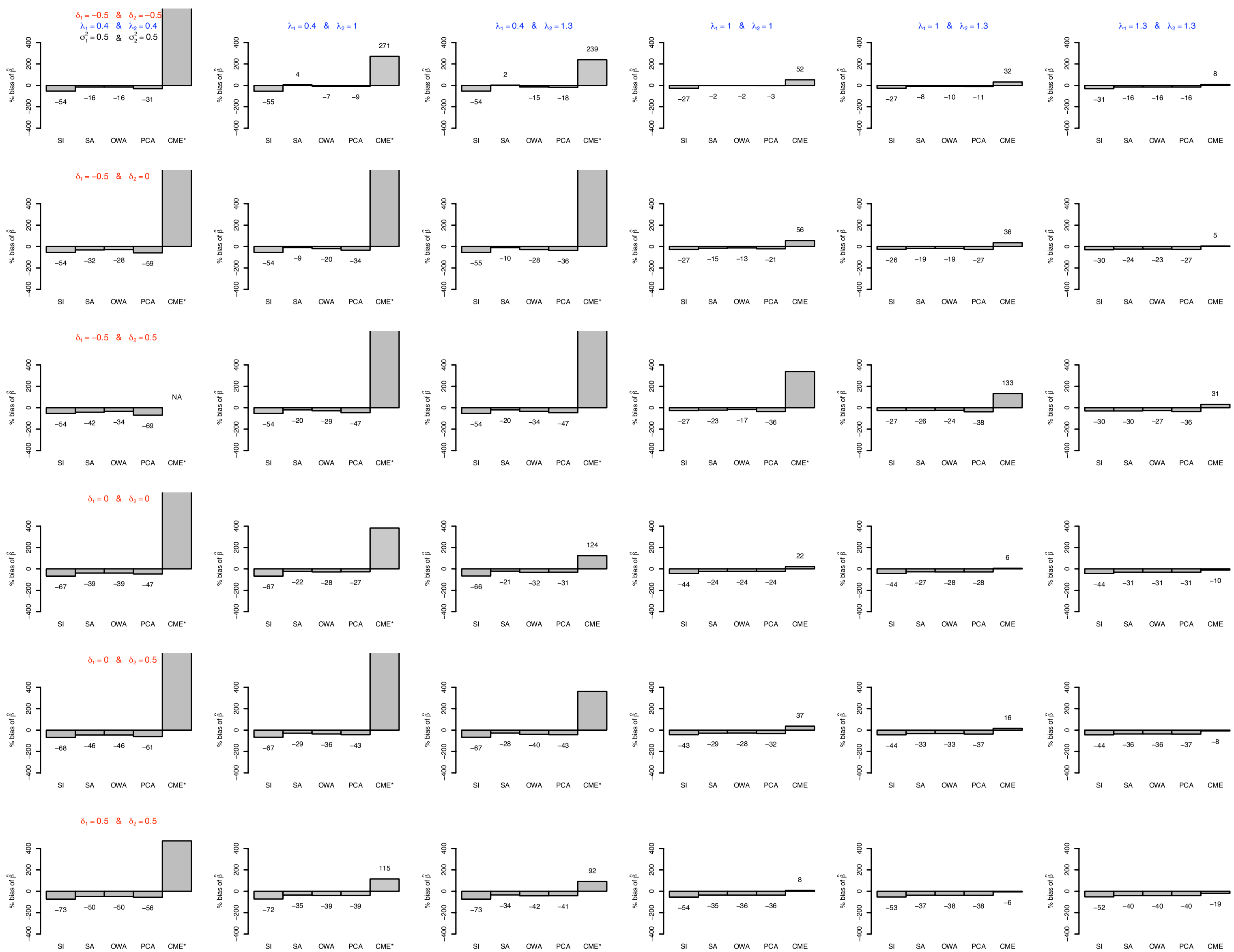
eFigure 16: Average Percent Bias of $\hat{\beta}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



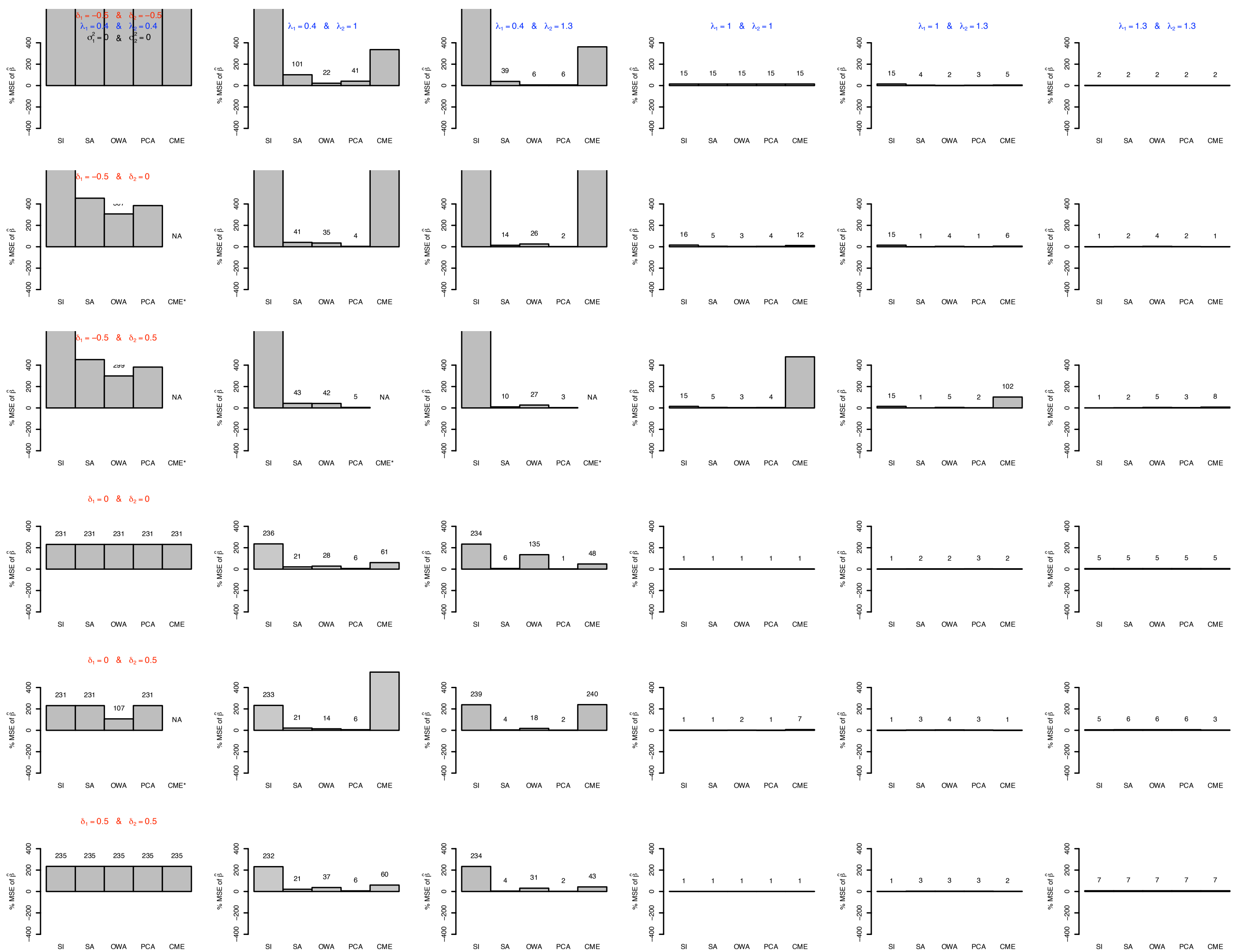
eFigure 17: Average Percent Bias of $\hat{\beta}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



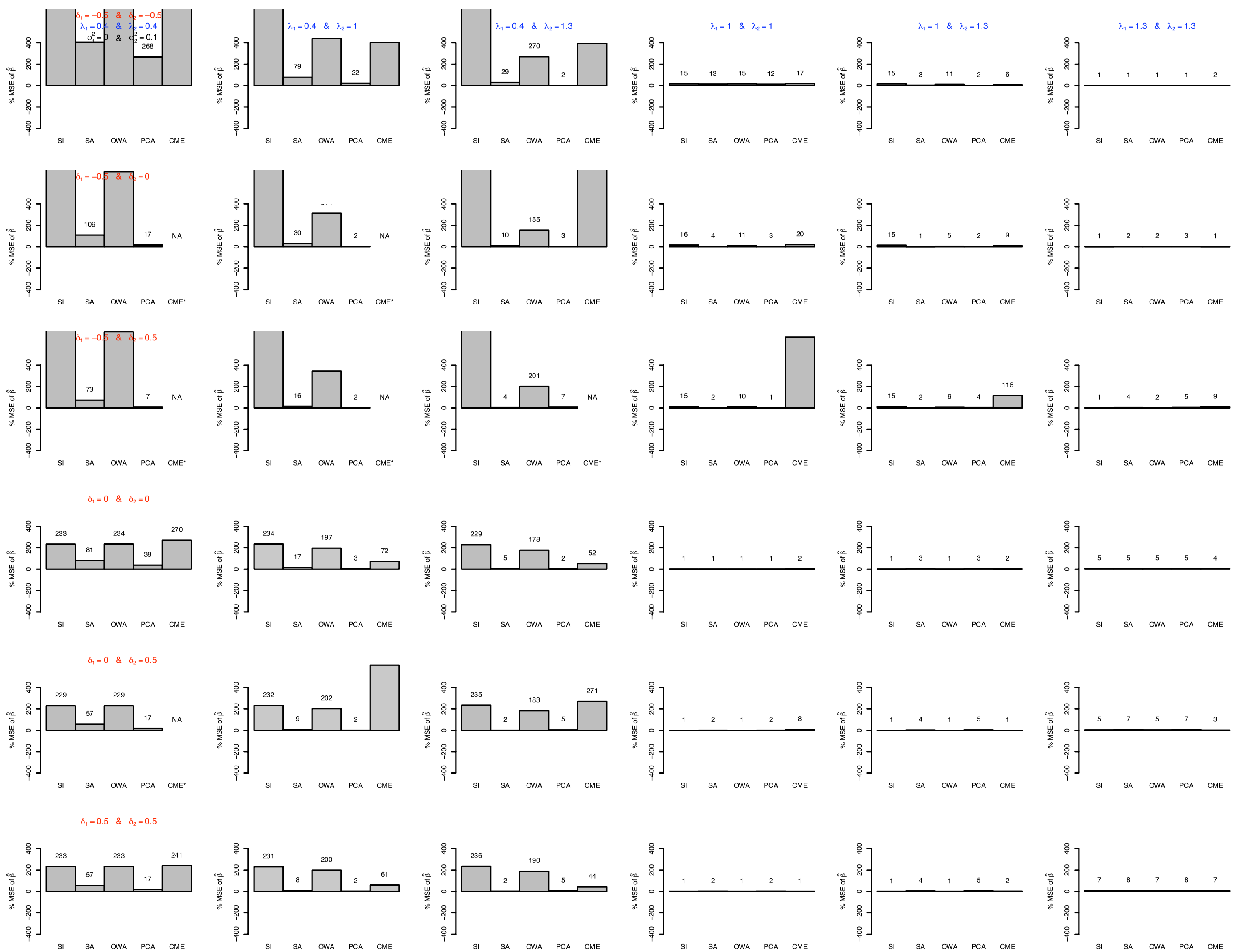
eFigure 18: Average Percent Bias of $\hat{\beta}$ for $\sigma_1^2 = 0.5$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



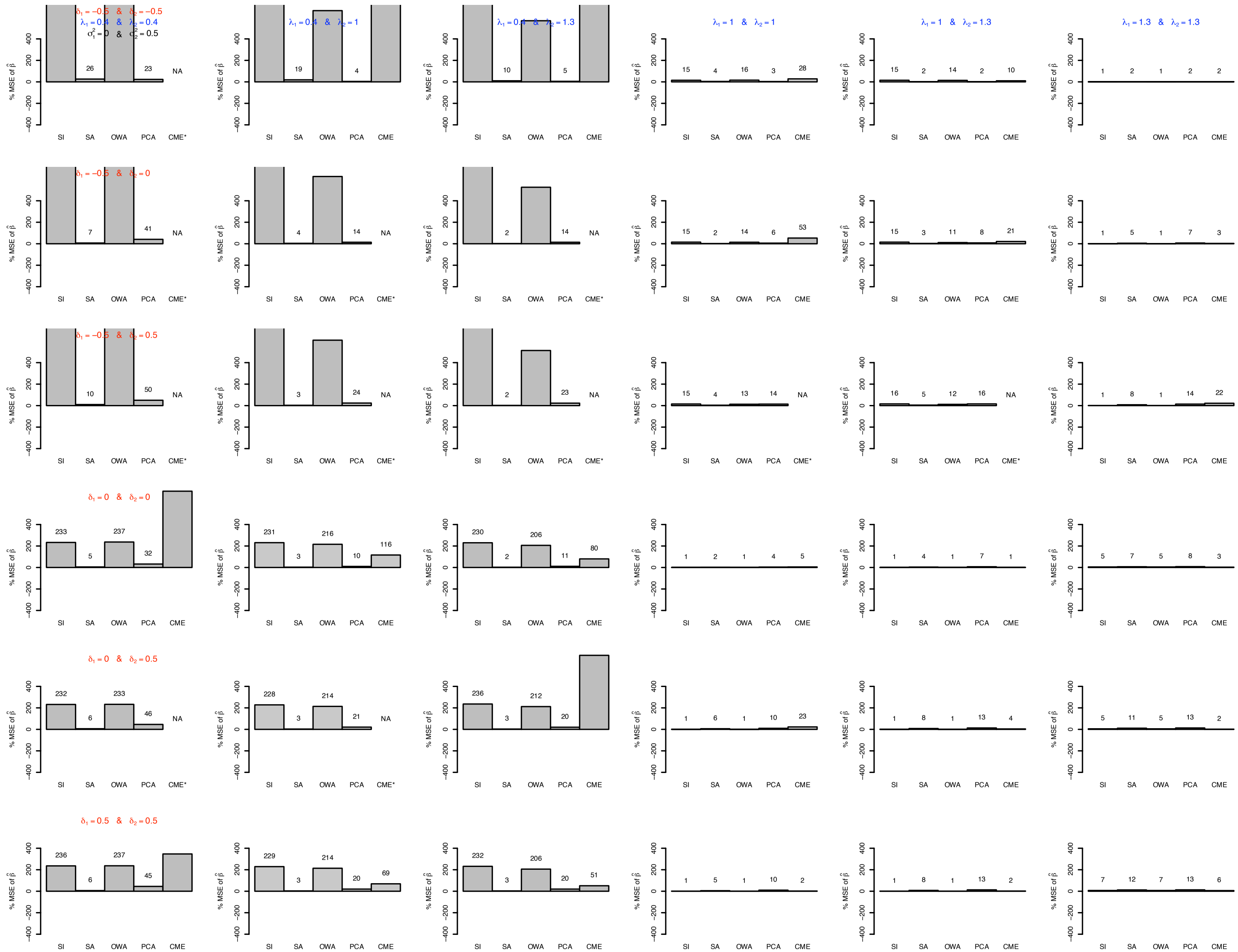
eFigure 19: Average Percent Mean Squared Error (MSE) of $\hat{\beta}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



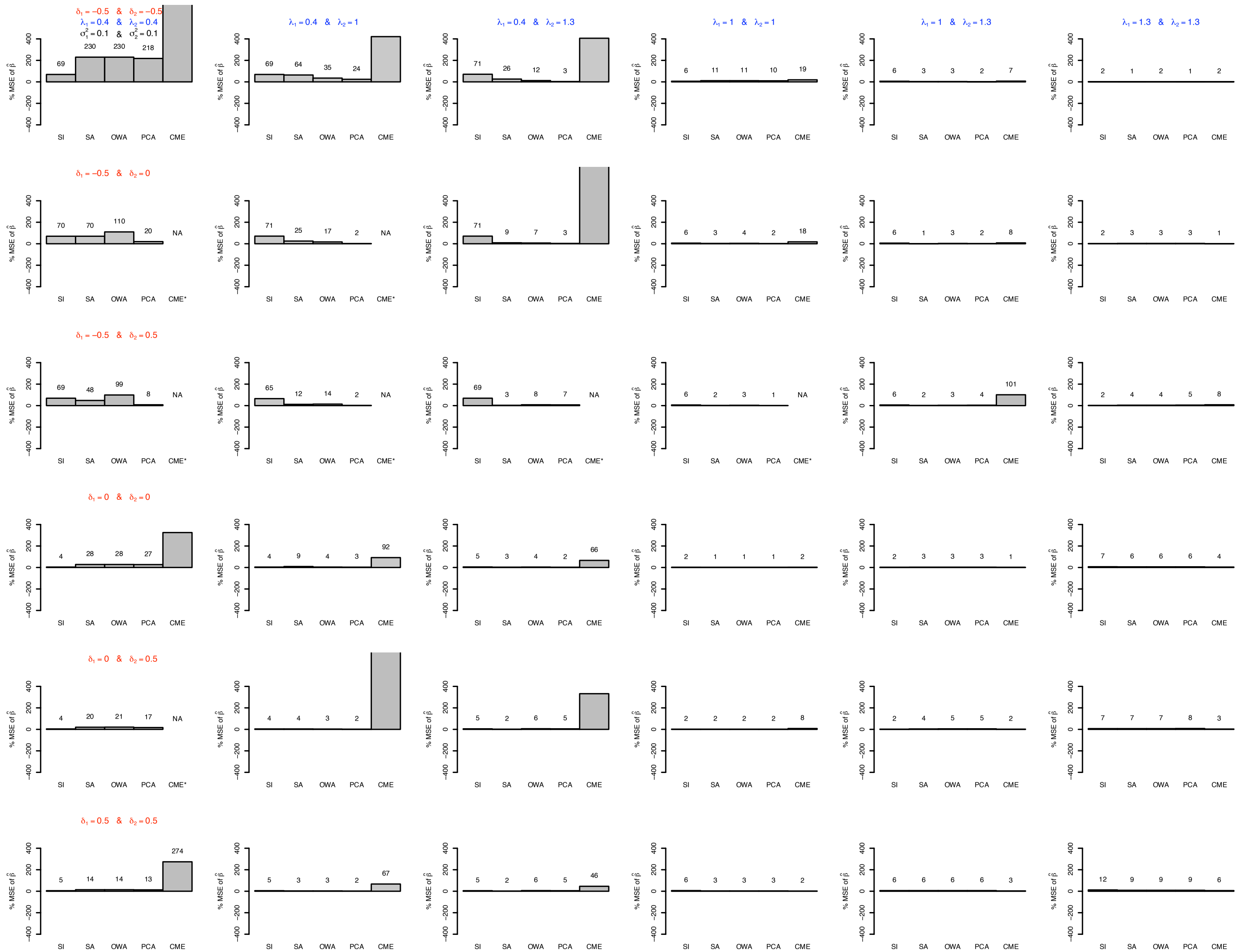
eFigure 20: Average Percent Mean Squared Error (MSE) of $\hat{\beta}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



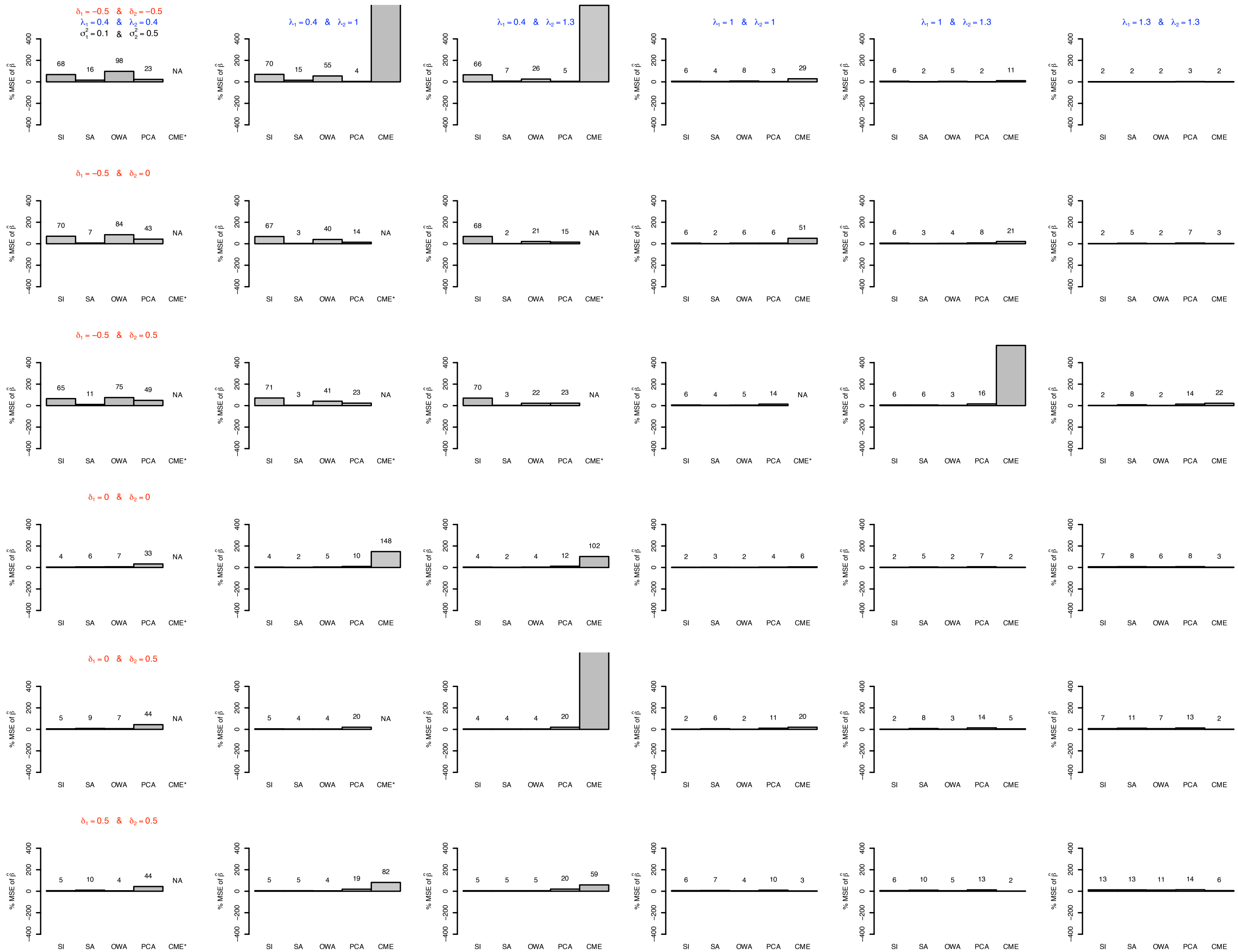
eFigure 21: Average Percent Mean Squared Error (MSE) of $\hat{\beta}$ for $\sigma_1^2 = 0$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



eFigure 22: Average Percent Mean Squared Error (MSE) of $\hat{\beta}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.1$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



eFigure 23: Average Percent Mean Squared Error (MSE) of $\hat{\beta}$ for $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.



eFigure 24: Average Percent Mean Squared Error (MSE) of $\hat{\beta}$ for $\sigma_1^2 = 0.5$ and $\sigma_2^2 = 0.5$ and Varying Values of δ_1, δ_2, λ_1, and λ_2, By Method.