

```

data {
  int<lower=0> N;
  // number of observations
  int<lower=0> Npat;
  // number of individuals
  real<lower=0> y[N];
  // outcome data
  real<lower=0> age[N];
  // explanatory variable (age) data
  int<lower=1,upper=Npat> id[N];
  // patient id for each observation
  vector<lower=0,upper=0>[4] zeros4;
  // mean
  // vector for random effects distribution
  int<lower=0> betakp_lower;
  // lower bound for (prior on) mean knot point
  int<lower=0> betakp_upper;
  // upper bound for (prior on) mean knot point

  int<lower=0> Npred;
  // number of predicted observations
  int<lower=0> Npat_pred;
  // number of patients to predict observations for
  real<lower=0> age_pred[Npred];
  // explanatory variable (age) for prediction
  int<lower=1,upper=Npat> id_pred[Npred];
  // patient id for each predicted observation
}

parameters {
  vector[3] beta;
  // fixed effects, intercept and slopes
  real<lower=betakp_lower,upper=betakp_upper> betakp; // fixed
  // effects, knotpoint (bounding specified to help convergence)
  vector<lower=0>[4] u_sd;
  // level 2 error sd (sds of the random effects u[j])
  real<lower=0> y_sd;
  // level 1 error sd
  vector[4] u[Npat];
  // random effects (level 2) errors
  cholesky_factor_corr[4] L_u_Corr;
  // cholesky factor for the random effects correlation matrix
}

transformed parameters {
  vector[4] alpha[Npat];
  // random effects
  real y_mu[N];
  // mean parameter based on regression equation

  //=====
  // calculate random effects
  //=====

  for (i in 1:Npat) {

```

```

        for (k in 1:3) alpha[i,k] <- beta[k] + u[i,k];
        alpha[i,4] <- betakp + u[i,4];
    }

    //=====
    // regression equation
    //=====

    for (j in 1:N) {
        if (age[j] < alpha[id[j],4])
            y_mu[j] <- alpha[id[j],1] + alpha[id[j],2] * (age[j] -
alpha[id[j],4]);
        else
            y_mu[j] <- alpha[id[j],1] + alpha[id[j],3] * (age[j] -
alpha[id[j],4]);
    }
}

model {

    //=====
    // priors
    //=====

    beta[1] ~ normal(20, 20); //
prior: fixed effect, intercept
    beta[2] ~ normal(0, 4);
    // prior: fixed effect, slope before knot
    beta[3] ~ normal(0, 4);
    // prior: fixed effect, slope after knot
    betakp ~ uniform(betakp_lower,betakp_upper); // prior: fixed
effect, knot point

    u_sd[1] ~ cauchy(0,5); //
prior: random effect sd, intercept
    u_sd[2] ~ cauchy(0,5); //
prior: random effect sd, slope before knot
    u_sd[3] ~ cauchy(0,5); //
prior: random effect sd, slope after knot
    u_sd[4] ~ cauchy(0,5); //
prior: random effect sd, knot point

    y_sd ~ cauchy(0,5);
    // prior: level 1 error sd

    L_u_Corr ~ lkj_corr_cholesky(1); // prior:
cholesky factor for random effects correlation matrix

    // NB. this prior is the "lkj correlation distribution" with
shape parameter 1

    // which is equivalent to a uniform distribution over the
possible correlation

```

```

// matrices (where a shape parameter > 1 would have resulted
in an upside down

// U-shaped distribution with the mode being located at the
identity matrix)

//=====
// random effects distribution
//=====

for (i in 1:Npat) u[i] ~ multi_normal_cholesky(zeros4,
diag_pre_multiply(u_sd, L_u_Corr)); // NB. the second parameter here
is the cholesky factor L

//
(for the correlation matrix). It only uses the sd rather

//
than the variances since Sigma = L*L'

//=====
// model likelihood
//=====

y ~ normal(y_mu, y_sd);
// likelihood for the observed data

}

generated quantities {
  real y_pred[Npred];
  // predicted outcome
  real y_mu_pred[Npred]; //
predicted mean
  corr_matrix[4] u_Corr; //
random effects correlation matrix
  matrix[4,4] u_Sigma; //
random effects covariance matrix
  vector[4] alpha_tosave[Npat_pred]; // monitor
random effects for a subset of patients only (for plotting predictions)
and do not monitor 'alpha' in the model above (since it consumes too much
memory!)

//=====
// predicted mean outcome using regression equation
//=====

for (i in 1:Npat_pred) {
  alpha_tosave[i] <- alpha[i];
}

for (j in 1:Npred) {
  if (age_pred[j] < alpha[id_pred[j],4])

```

```

        y_mu_pred[j] <- alpha[id_pred[j],1] +
alpha[id_pred[j],2] * (age_pred[j] - alpha[id_pred[j],4]);
        else
        y_mu_pred[j] <- alpha[id_pred[j],1] +
alpha[id_pred[j],3] * (age_pred[j] - alpha[id_pred[j],4]);

        y_pred[j] <- normal_rng(y_mu_pred[j], y_sd);
    }

    //=====
    // recover the correlation and covariance matrices
    // using the cholesky factor of the correlation matrix
    //=====

    u_Corr <- multiply_lower_tri_self_transpose(L_u_Corr);          //
correlation matrix: u_Corr = L_u_Corr * L_u_Corr'
    u_Sigma <- quad_form_diag(u_Corr, u_sd);
    // covariance matrix: u_Sigma = diag(u_sd) * u_Corr * diag(u_sd)

}

```