eAppendix for "Bayesian piecewise linear mixed models with a random change point: an application to BMI rebound in childhood"

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1. Model implementation

We adopted a Bayesian approach to estimation, implemented in the software Stan [1]. Stan provides a similar modelling language to other Bayesian software packages, such as WinBUGS [2] and JAGS [3], but has some important benefits for our model fitting. In particular Stan uses a specific implementation of a Hamiltonian Monte Carlo algorithm for obtaining the Markov chain Monte Carlo (MCMC) samples. This results in faster convergence and lower autocorrelation between subsequent MCMC samples than would likely be achieved using a random walk MCMC algorithm such as Gibbs sampling (which is the sampling algorithm primarily used by WinBUGS and JAGS).

Further, Stan allows sampling from an “LKJ” correlation matrix distribution, which we use as a prior distribution for the random effects correlation matrix in our model [4]. The “LKJ” correlation matrix distribution allows us to specify a prior distribution for the random-effects correlation matrix **R** and then independently specify a prior distribution for each of the standard deviation terms ($\sigma_{u_k}; k = 1,2,3,4$) for the random effects, thereby allowing us to estimate these two components separately in a much more intuitive way [5]. In this way we do not need to specify a prior distribution for the variance-covariance matrix $\mathbf{\Sigma}$ since it can be calculated explicitly using the correlation matrix **R** and the standard deviations for each of the random effects.

In the plain text file entitled “stancode_randomchangecorr.stan” we provide the Stan model code for fitting the random change point model described in the main manuscript, where the random change point is correlated with all other individual-level random effects in the model. On request the authors are happy to provide the Stan model code for the simpler alternative models, namely one which assumes a fixed (common) change point for all individuals or one which assumes the random change point is independent from the other individual-level random effects.

In the R script file entitled "R_example_fitting_the_random_change_point_model_in_Stan.R" we have provided R code which allows the user to create a single simulated dataset (similar to the ALSPAC dataset used in the main analysis of our paper) and then fit the random change point model to this simulated dataset using Stan. Several R packages will need to be installed first, and these are listed at the top of the R script file with details on how they can be installed. Comments are also provided in the R script file which should explain the main steps required to fit the model, as well as looking at some diagnostics (e.g. MCMC trace plots, Hamiltonian Monte Carlo sampler diagnostics) and inference (e.g. model parameter estimates, plots of fitted longitudinal trajectories for a subset of individuals).

2. Further details on prior distributions and model estimation

In this section we provide further detail on the prior distributions and estimation of the model described in the main manuscript. Following the advice of Gelman we use weakly informative prior distributions [6]. Our prior distributions are not intended to provide support to specific parameter values based on prior knowledge or experience, but are intended to reduce support for parameter values which seem biologically implausible. For the fixed intercept β_{10} we use a Cauchy distribution with location 20 and scale 20. For each of the fixed slope parameters β_{20} and β_{30} we use a Cauchy distribution with location 0 and scale 4. For the fixed change point ω_0 we use a bounded uniform prior on the range (3,9) years of age. For the residual standard deviation of the observed BMI measurements σ_y we use a half-Cauchy distribution with location 0 and scale 5. For the random effects correlation matrix **R** we use the "LKJ" correlation matrix distribution parameterised in terms of its Cholesky factor and with shape parameter equal to 1. This prior distribution provides a uniform prior over all possible correlation matrices (see Section 50.2 of [7] for details). For the univariate standard deviations ($\sigma_{u_k}; k = 1,2,3,4$) of each of the random effects we use a half-Cauchy prior with location 0 and scale 5.

For fitting the model we randomly generate three sets of disparate initial values (known as "chains"). We obtain 1,500 MCMC sample iterations from each of the 3 chains, preceded by a discarded "warm-up" period of 500 MCMC iterations. Convergence is assessed using the potential scale reduction statistic [7, 8] as well as visually inspecting trace plots of the MCMC samples.

3. Residual plots from the fitted model

In this section we present some residual diagnostics for the random change point model fitted to the ALSPAC data (the main methods and results for the analysis are described in the main manuscript).

Figures S1 and S2 show, for females and males respectively, a series of L kernel density plots of the standardised residuals from the fitted model. That is, the l^{th} ($l = 1, \dots, L$) kernel density plot shows the distribution of the standardised residuals

$$\hat{r}_{ij}{}^{(l)} = \frac{y_{ij} - \hat{\mu}_{ij}{}^{(l)}}{\hat{\sigma}_y{}^{(l)}}$$

calculated using the l^{th} draw of parameters from the posterior distribution. We set $L = 9$ draws for plotting the distribution of the residuals, however, it is worth noting that these L draws are chosen at random from the total number of draws (i.e., sample iterations) taken when fitting the model. Figures S3 and S4 show the same residuals as Figures S1 and S2, but in this case the residuals are plotted against the time variable, which is age (in years).

There does not appear to be any obvious skewness in the distribution of the residuals, nor does there appear to be any obvious patterns in the residuals when plotted against the observation time. The variability of the residuals appears to be constant over time. Of the total number of BMI observations – 38,684 for females and 39,366 for males – approximately 1% had a standardised residual outside the range (-3, +3) and approximately 4% had a standardised residual outside the range (-2, +2), in line with what would be expected under a normal distribution, and this result was similar for both genders and across all 9 draws.

4. Simulation study

In this section we present a simulation study in which we compare the performance of our random change point model to several alternative models. In generating data for our simulation study we assume that there is true underlying heterogeneity between individuals in terms of when BMI rebound occurs. Our simulation study has two main objectives. First, to compare our random change point model to other piecewise linear mixed models which are potentially simpler to fit, but do not allow for this heterogeneity and are therefore misspecified. Second, to assess how reliably our random change point model estimates the mean timing of BMI rebound compared with a more complex alternative model based on fractional polynomials.

Model specification

Similar to the model presented in the main manuscript, the first three alternative models we consider are piecewise linear mixed models with a single change point. The first two models assume a fixed (common) change point for all individuals, whilst the third model assumes a random change point that is independent from the other individual-level random effects.

Taking the model defined in equation (1) of the main manuscript, we consider the following three specifications for the change point

$$\text{Model 1:} \qquad \omega_i = \text{c}$$

$$\text{Model 2:} \qquad \omega_i = \omega \tag{1}$$

$$\text{Model 3:} \qquad \omega_i = \omega_0 + u_{4i}$$

Models 1 and 2 assume the change point is the same for all individuals, however, model 1 assumes the change point is a constant value c which is known *a priori* whereas model 2 assumes the change point is a fixed parameter ω which is estimated from the data. For models 1 and 2 we assume the individual-level random effects u_{1i}, u_{2i} and u_{3i} are multivariate normally distributed with mean zero and a 3x3 unstructured variance-covariance matrix (note that u_{4i} does not exist for these models). Aside from the different specification for the change point and the resulting reduction in dimensionality of the random effects distribution (3-dimensions rather than 4-dimensions) models 1 and 2 are otherwise the same as the model described in Section 2.

Model 3 is the random change point model, however, we further specify this model in two ways. The first is to estimate u_{4i} independently of the other individual-level random effects, that is

$$\begin{bmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \end{bmatrix} \sim MVN\left(\mathbf{0}\,, \qquad \mathbf{\Sigma_{3x3}} = \begin{bmatrix} \sigma_{u_1}^2 & \sigma_{u_1u_2} & \sigma_{u_1u_3} \\ \sigma_{u_1u_2} & \sigma_{u_2}^2 & \sigma_{u_2u_3} \\ \sigma_{u_1u_3} & \sigma_{u_2u_3} & \sigma_{u_3}^2 \end{bmatrix} \right) \tag{6}$$

and $u_{4i} \sim N\left(0,\ \sigma_{u_4}^2\right)$; we label this 'model 3a'. The second is to use the fully unstructured variance-covariance matrix for the random effects distribution as described in Section 2; we label this 'model 3b'.

The final alternative model we consider (model 4) is of a different form. It is a linear mixed model that allows for flexibility in the longitudinal BMI trajectories through the use of fractional polynomials. Using the ALSPAC data (which is described in the main manuscript) we identified the best fitting fractional polynomial model, allowing for a maximum degree of

2 and considering the set of powers (-1, -0.5, 0, 0.5, 1, 2, 3). For both the male and female ALSPAC data the best fitting model based on the Bayesian Information Criterion (BIC) was degree 2 with powers (1, 1). Since repetition of powers in fractional polynomial models corresponds to an interaction term with the natural logarithm of the variable, this leads to a model of the form

$$
\begin{gathered}
y_{ij} \sim N\left(\mu_{ij}, \sigma_y^2\right) \\
\mu_{ij} = \alpha_{1i} + \alpha_{2i} t_{ij} + \alpha_{3i} t_{ij} \ln(t_{ij}) \\
\begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \\ \alpha_{30} \end{bmatrix} + \begin{bmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \end{bmatrix}
\end{gathered} \tag{7}
$$

where the individual-level random effects u_{1i}, u_{2i}, u_{3i} are multivariate normally distributed as specified in equation (6). Note that the fixed effect parameters α_{10}, α_{20}, and α_{30} in equation (7) are not easily interpretable.

The fractional polynomial model in equation (7) provides a smooth non-linear trajectory over time. Since it does not contain a parameter corresponding to the timing of BMI rebound, we must derive one. We take the timing of BMI rebound to be the turning point in the fractional polynomial model, which we calculate analytically. The estimate for the mean timing of BMI rebound is taken to be the turning point for the estimated marginal BMI trajectory, that is,

$$
\hat{\omega}_0 = \exp\left(\frac{-(\hat{\alpha}_{20} + \hat{\alpha}_{30})}{\hat{\alpha}_{30}}\right) \tag{8}
$$

The formula in equation (8) leads to a local minimum when $\hat{\alpha}_{30}$ is greater than zero. In our simulation study the estimate for $\hat{\alpha}_{30}$ was always positive, and therefore the formula in equation (8) provided a sensible estimate of the timing of BMI rebound.

It is worth noting that the corresponding formula for the turning point of the subject-specific fractional polynomial trajectory, that is,

$$\widehat{\omega}_{0i} = \exp\left(\frac{-(\hat{\alpha}_{2\mathrm{i}} + \hat{\alpha}_{3\mathrm{i}})}{\hat{\alpha}_{3i}}\right) \tag{9}$$

similarly leads to a local minimum when $\widehat{\alpha}_{3\mathrm{i}}$ is greater than zero. In some cases however the subject-specific estimate $\widehat{\alpha}_{3\mathrm{i}}$ may be negative due to an individual with an extreme draw from the random effects distribution, in which case the formula in equation (9) will lead to a local maximum that cannot be used to calculate an estimate of the subject-specific timing of BMI rebound.

Data

We simulated data from two models. First, we generated BMI data under our random change point model, defined in equation (1) of the main manuscript. Second, we generated data under the fractional polynomial model, defined in equation (7) above. The true parameter values used in the simulation study are described below. For both data generating models, the true parameter values were guided by parameter estimates obtained from an analysis of the ALSPAC data.

The number of observations n_i for the ith child was randomly generated to be between 5 and 25 (generated as the integer component of a $Unif(5,25)$ random variable). Observation times were assumed to be uniformly distributed between ages 1 and 15 years, $t_{ij} \sim Unif(1, 15)$.

For data generated under the random change point model, we simulated random parameters β_{1i}, β_{2i}, β_{3i} and ω_i for each child according to the multivariate normal distribution with mean vector $\boldsymbol{\mu} = (15, -0.4,\ 0.6,\ 6.5)$ and one of the following variance-covariance matrices (with corresponding correlation matrix)

Scenario 1 ("weak" correlation):

$$\mathbf{\Sigma} = \begin{bmatrix} 1.440 & 0.072 & 0.072 & -0.168 \\ 0.072 & 0.040 & -0.012 & 0.028 \\ 0.072 & -0.012 & 0.090 & -0.042 \\ -0.168 & 0.028 & -0.042 & 1.960 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 1 & 0.3 & 0.2 & -0.1 \\ 0.3 & 1 & -0.2 & 0.1 \\ 0.2 & -0.2 & 1 & -0.1 \\ -0.1 & 0.1 & -0.1 & 1 \end{bmatrix}$$

Scenario 2 ("moderate" correlation):

$$\mathbf{\Sigma} = \begin{bmatrix} 1.440 & 0.072 & 0.072 & -0.504 \\ 0.072 & 0.040 & -0.012 & 0.084 \\ 0.072 & -0.012 & 0.090 & -0.126 \\ -0.504 & 0.084 & -0.126 & 1.960 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 1 & 0.3 & 0.2 & -0.3 \\ 0.3 & 1 & -0.2 & 0.3 \\ 0.2 & -0.2 & 1 & -0.3 \\ -0.3 & 0.3 & -0.3 & 1 \end{bmatrix}$$

Scenario 3 ("strong" correlation):

$$\mathbf{\Sigma} = \begin{bmatrix} 1.440 & 0.072 & 0.072 & -0.84 \\ 0.072 & 0.040 & -0.012 & 0.14 \\ 0.072 & -0.012 & 0.090 & -0.21 \\ -0.84 & 0.14 & -0.21 & 1.960 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 1 & 0.3 & 0.2 & -0.5 \\ 0.3 & 1 & -0.2 & 0.5 \\ 0.2 & -0.2 & 1 & -0.5 \\ -0.5 & 0.5 & -0.5 & 1 \end{bmatrix}$$

These three scenarios differ in terms of the magnitude of the true underlying correlation between the random change point and the other individual-level random effects. Given that our random change point model (model 3b) incorporates an unstructured variance-covariance matrix for the random effects, and our simpler alternative change point models (models 1, 2, 3a) do not, we wished to assess whether this correlation affected the bias in the obtained parameter estimates. Observed BMI measurements for each child were then generated under the model in equation (1) of the main paper, setting the standard deviation of the residual error terms (σ_y) equal to 1.

For data generated under the fractional polynomial model, we simulated random parameters α_{1i}, α_{2i}, and α_{3i} for each child according to the multivariate normal distribution with mean vector $\boldsymbol{\mu} = (20, -2.1,\ 0.73)$ and the variance-covariance matrix (with corresponding correlation matrix)

$$\boldsymbol{\Sigma} = \begin{bmatrix} 4 & -1.66 & 0.462 \\ -1.66 & 1 & -0.288 \\ 0.462 & -0.288 & .09 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 1 & -0.83 & 0.77 \\ -0.83 & 1 & -0.96 \\ 0.77 & -0.96 & 1 \end{bmatrix}$$

We calculate the true mean timing of BMI rebound for data generated under the fractional polynomial model as the turning point for the true underlying marginal BMI trajectory, that is, using equation (8) we have $\omega_0 = \exp\left(-\frac{(-2.1+0.73)}{0.73}\right) = 6.532$ years.

For all scenarios in the simulation study we generated 100 datasets, with each dataset consisting of the observed BMI measurements for either N = 50, 100 or 150 children. For each of the model parameters we report the mean (across the 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. Here we define "bias" as the difference between the estimated posterior mean for the parameter and the true parameter value.

Results based on the random change point data generating model

We first discuss the results based on the random change point data generating model. Tables S1 through S3 show results under scenarios 1 through 3, respectively, using a sample size of N = 100 children. Tables S4 through S6 show the results using a sample size of N = 50 children, and Tables S7 through S9 show the results using a sample size of N = 150 children. We focus on the findings from the simulation study using N = 100 children, since the different sample sizes produced similar results. We do note that the level of bias in the covariance and correlation parameters under the random change point model (model 3b) appeared to increase slightly in simulations with the smallest sample size (N = 50 children).

Compared with the fixed change point models (models 1 and 2) the random change point models (models 3a and 3b) generally resulted in smaller bias and higher coverage for all parameters in the model under all three scenarios. One exception, however, was the average change point ω_0 which exhibited similar levels of bias (albeit a small bias) under models 2, 3a and 3b, although the coverage was generally higher under the random change point models (models 3a and 3b). For the fixed change point models a reasonably large relative bias was observed for the average pre-change slope β_{20}, which was underestimated, and the standard deviation estimates for the random effects σ_{u_1}, σ_{u_2} and σ_{u_3}, which were all overestimated. The covariance and correlation parameter estimates for the random effects were all severely biased under the fixed change point model. For the random change point model the only parameters which exhibited any potentially concerning level of bias were the standard deviation for the random pre-change slope σ_{u_2} and the covariance and correlation parameters for the random effects, however these were dramatically less biased than under the fixed change point model (for those parameters where comparisons could be made).

Increasing the true correlation between the random change point and the other individual-level random effects (for example scenario 3 vs scenario 1) resulted in increasing bias for the model which did not account for this correlation (model 3a). While the covariance and correlation parameters for the random effects were most severely affected, the fixed effect regression coefficients also showed larger relative bias. For example, under scenario 1 (weak correlation) the relative bias for the covariance parameters in model 3a ranged between -27.3% and +10.4%, whereas under scenario 3 (strong correlation) the relative bias for the covariance parameters in model 3a ranged between -158.8% and +94.1%. Under scenario 1 (weak correlation) the relative bias for the fixed effect regression coefficients in model 3a ranged between -0.6% and +1.0%, whereas under scenario 3 (strong correlation) the relative bias for the fixed effect regression coefficients in model 3a ranged between -6.2% and +0.5%. On the other hand, the model with the unstructured variance-covariance matrix across all individual-level random effects (model 3b) showed stable and low levels of bias, as well as reasonable coverage, under all three scenarios.

The fractional polynomial model (model 4) severely underestimated the mean timing of BMI rebound, with the relative bias for ω_0 ranging between -19.9% (scenario 1) and -23.4% (scenario 3) and coverage probabilities of zero in all three scenarios. That is, the turning point for the marginal fractional polynomial model provided a biased (under)estimate of the mean change point.

Results based on the fractional polynomial data generating model

We now turn our attention to results based on the fractional polynomial data generating model. Tables S10 through S12 show the results for sample sizes of N = 100, 50 and 150 children, respectively. We focus on the findings from the simulation study using N = 100 children (Table S10) since, aside from the overall slight increase in bias which was associated with a smaller sample size, the three sample sizes produced similar results.

For these data, the fractional polynomial model (model 4) returned low levels of bias and high coverage (between 0.92 and 0.93) for the fixed effect parameters α_{10}, α_{20}, and α_{30}. The standard deviation estimates for the random effects, as well as the covariance and correlation parameter estimates for the random effects, were all slightly underestimated by the fractional polynomial model with relative bias ranging between -1.8% and -15.0% and coverage probabilities ranging between 0.86 and 0.93. The fractional polynomial model estimated the mean turning point with negligible bias (relative bias of 0.1%) and high coverage (0.94).

Importantly, there was little bias in the mean timing of BMI rebound when estimated using the change point models. The relative bias for ω_0 was -1.5%, -1.9% and -2.1% for models 2, 3a and 3b, respectively, when they were fitted to the fractional polynomial generated data. It is noteworthy that these estimates are dramatically less biased than the estimates for ω_0 obtained from fitting the fractional polynomial model (model 4) to data generated under the random change point model. The estimate for ω_0 achieved higher coverage (coverage = 0.92) under model 3b, than under model 3a (coverage = 0.80) or model 2 (coverage = 0.75).

References

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Figure S1. Kernel density plots of the standardised residuals for the random change point model, estimated using the female ALSPAC data. The 9 different kernel density plots represent 9 randomly chosen MCMC draws of the parameters from the joint posterior distribution.

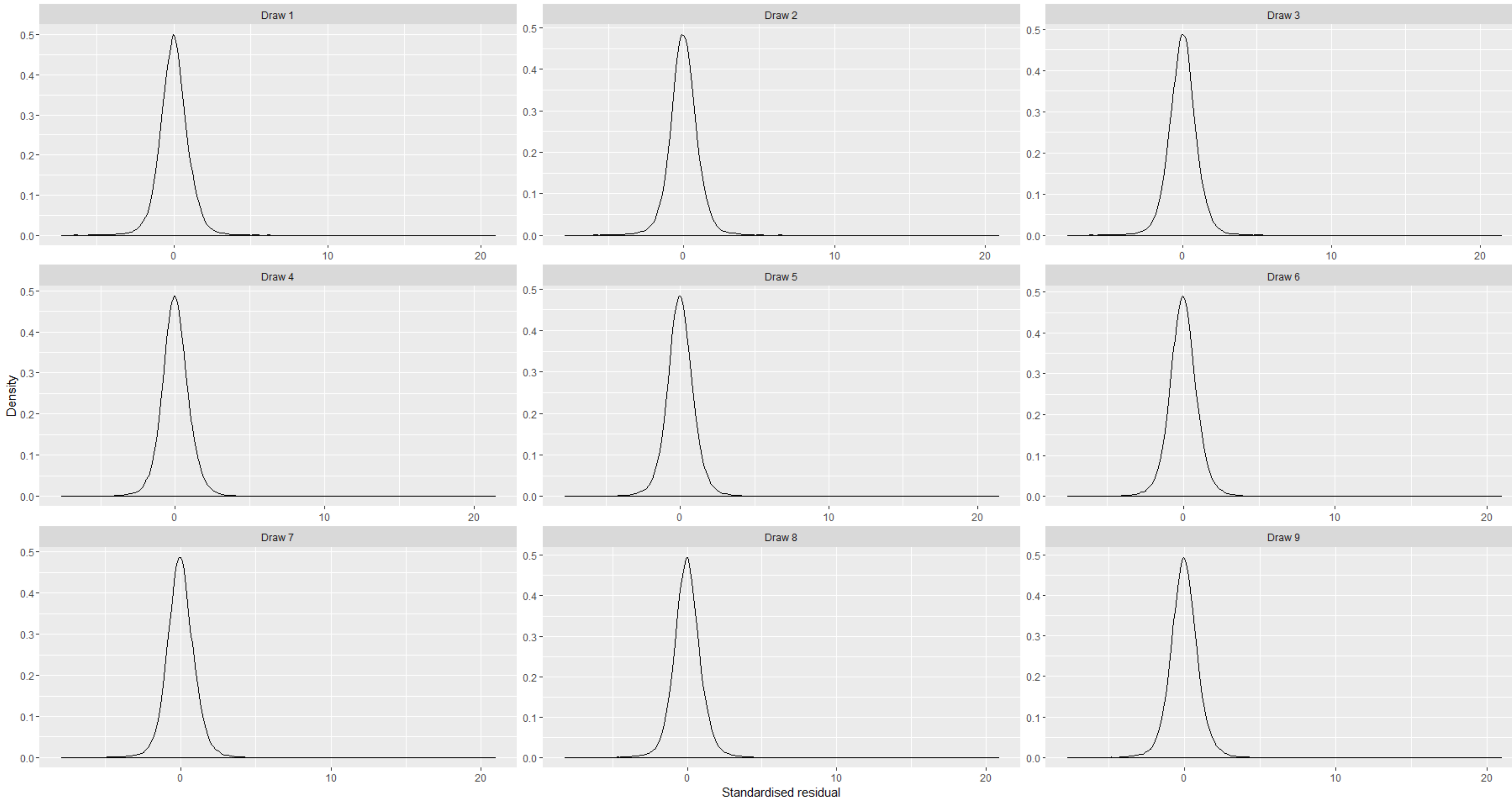

Figure S2. Kernel density plots of the standardised residuals for the random change point model, estimated using the male ALSPAC data. The 9 different kernel density plots represent 9 randomly chosen MCMC draws of the parameters from the joint posterior distribution.

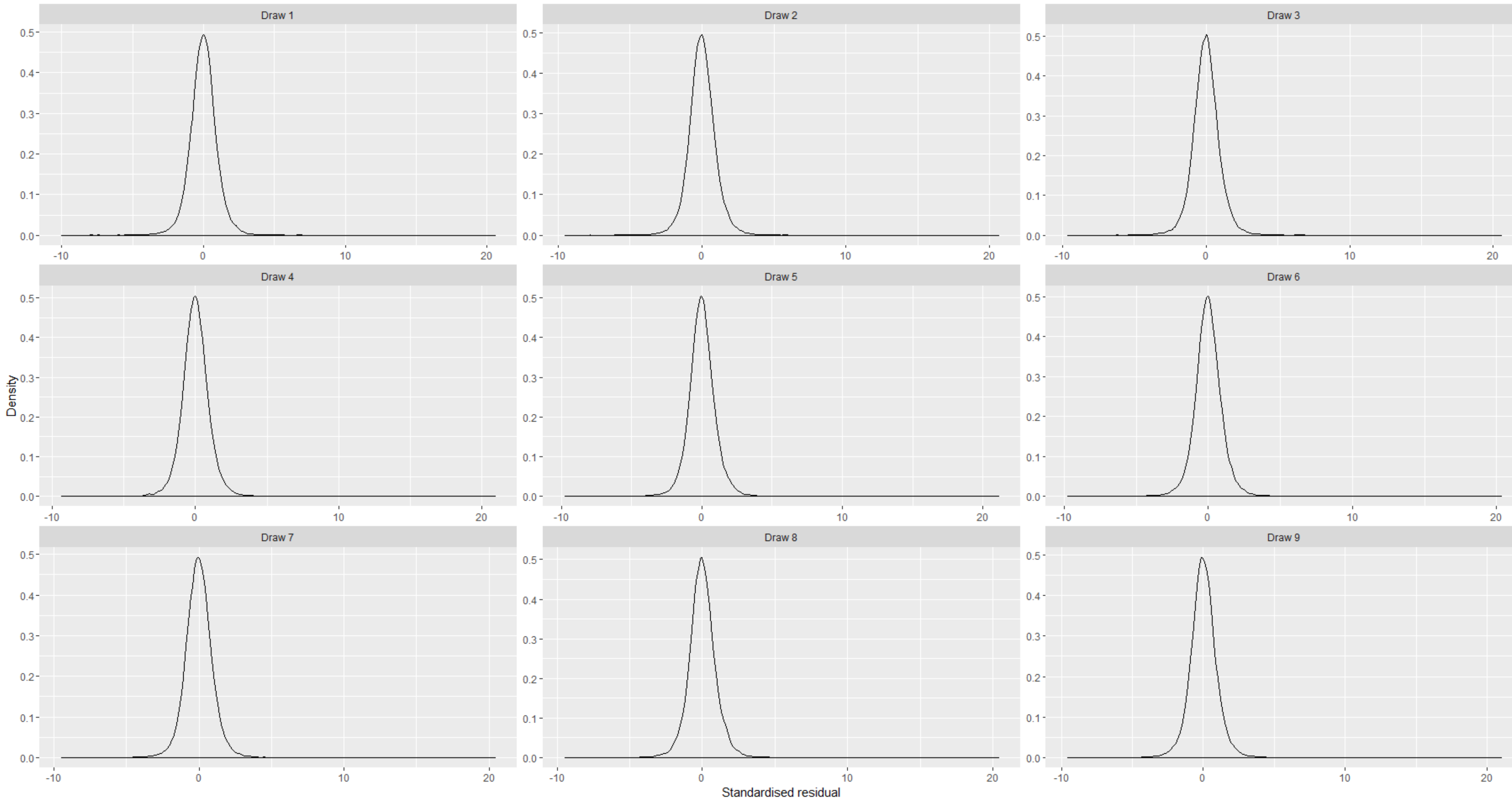

Figure S3. Scatterplots of the standardised residuals against observed age (in years), for the random change point model estimated using the female ALSPAC data. The 9 different scatterplots represent 9 randomly chosen MCMC draws of the parameters from the joint posterior distribution.

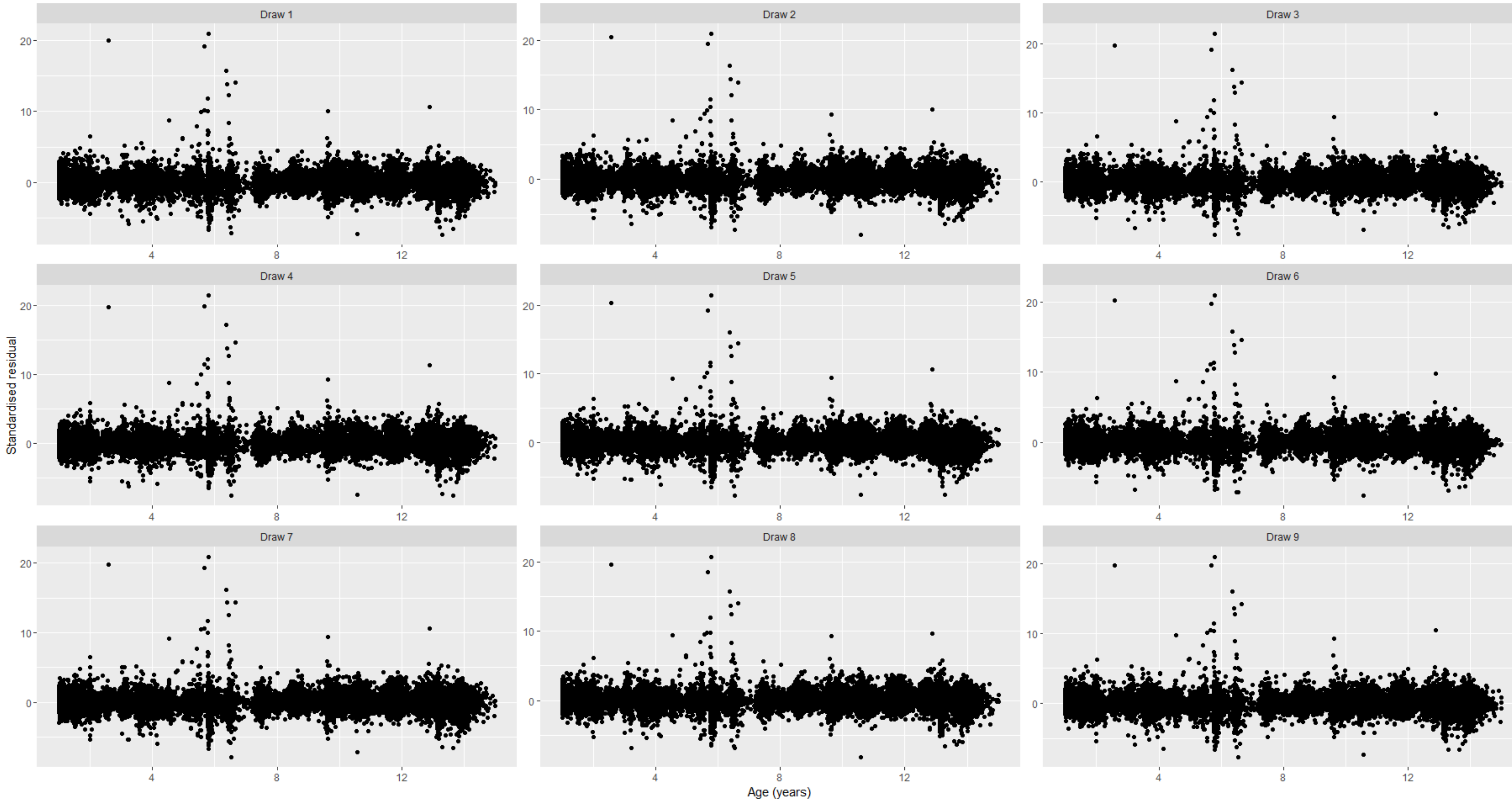

Figure S4. Scatterplots of the standardised residuals against observed age (in years), for the random change point model estimated using the male ALSPAC data. The 9 different scatterplots represent 9 randomly chosen MCMC draws of the parameters from the joint posterior distribution.

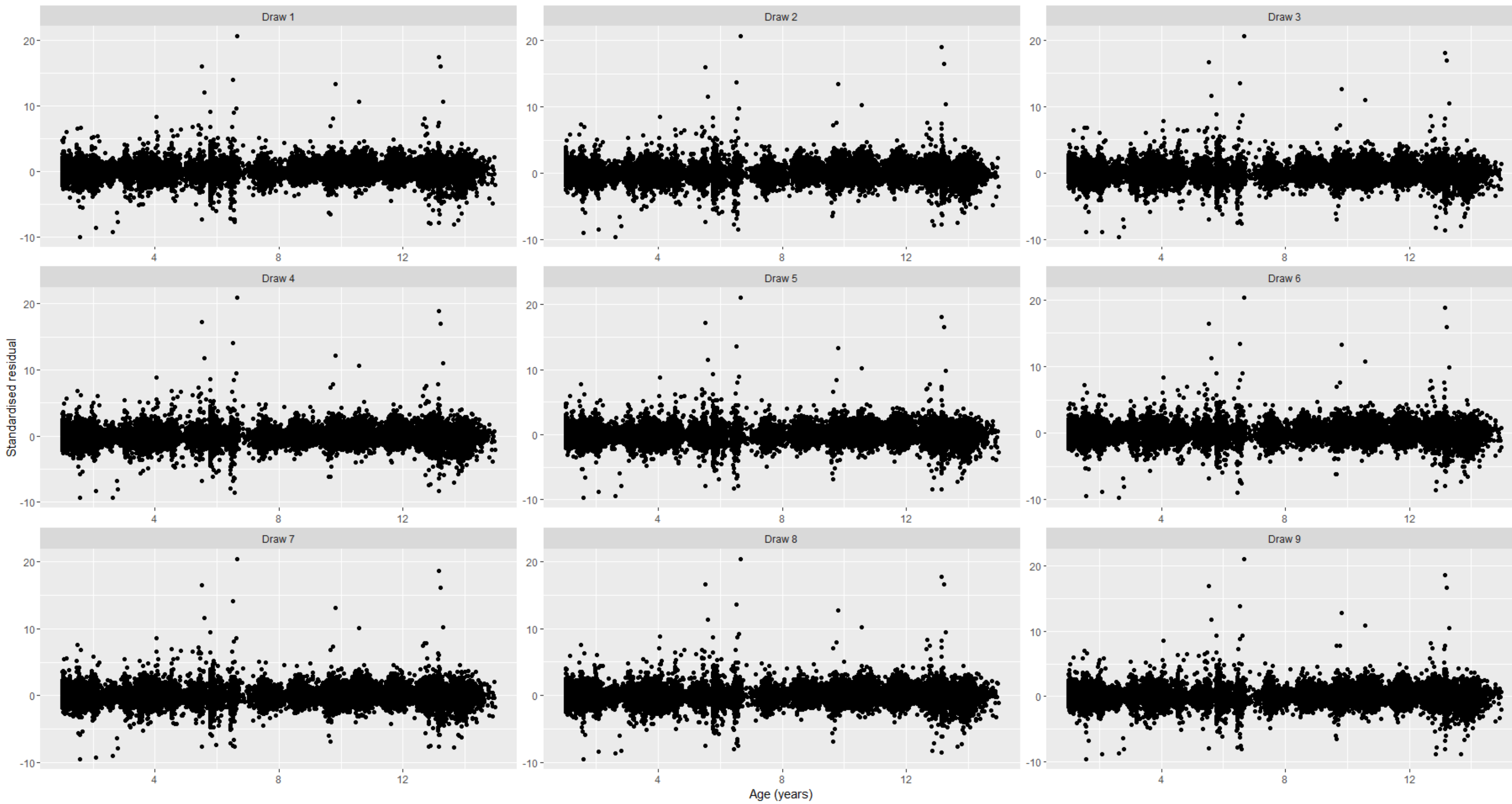

Table S1. Simulation study for change point generated data, under scenario 1 (weak correlation between the random change point and the other individual-level random effects) using a sample size of 100 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.276	1.8%	0.53	0.297	2.0%	0.52	0.016	0.1%	0.95	0.016	0.1%	0.96	na	na	na
β_{20}	-0.4	0.077	-19.2%	0.50	0.088	-22.0%	0.46	-0.004	1.0%	0.96	-0.009	2.2%	0.93	na	na	na
β_{30}	0.6	-0.035	-5.9%	0.87	-0.027	-4.5%	0.89	-0.004	-0.6%	0.94	-0.002	-0.3%	0.94	na	na	na
ω_0	6.5	na	na	na	0.103	1.6%	0.66	-0.131	-2.0%	0.89	-0.109	-1.7%	0.92	-1.291	-19.9%	0.00
σ_y	1	0.029	2.9%	0.68	0.028	2.8%	0.69	-0.001	-0.1%	0.94	0.000	0.0%	0.96	0.045	4.5%	0.46
σ_{u_1}	1.2	0.161	13.4%	0.72	0.179	14.9%	0.66	0.003	0.3%	0.94	-0.008	-0.7%	0.94	na	na	na
σ_{u_2}	0.2	0.112	55.9%	0.12	0.110	55.2%	0.10	-0.024	-12.1%	0.91	-0.019	-9.7%	0.96	na	na	na
σ_{u_3}	0.3	0.026	8.8%	0.89	0.028	9.4%	0.86	0.014	4.6%	0.97	0.014	4.6%	0.97	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	0.013	0.9%	0.95	0.063	4.5%	0.95	na	na	na
$\rho_{u_1u_2}$	0.3	0.173	57.8%	0.64	0.186	62.1%	0.52	-0.008	-2.5%	0.97	-0.096	-31.9%	0.95	na	na	na
$\rho_{u_1u_3}$	0.2	0.098	48.9%	0.90	0.103	51.6%	0.87	0.013	6.7%	0.96	-0.014	-7.1%	0.97	na	na	na
$\rho_{u_1u_4}$	-0.1	na	na	na	na	na	na	na	na	na	-0.037	37.0%	0.99	na	na	na
$\rho_{u_2u_3}$	-0.2	0.401	-200.7%	0.18	0.401	-200.6%	0.17	0.043	-21.4%	0.99	0.055	-27.5%	0.98	na	na	na
$\rho_{u_2u_4}$	0.1	na	na	na	na	na	na	na	na	na	-0.089	-88.7%	0.99	na	na	na
$\rho_{u_3u_4}$	-0.1	na	na	na	na	na	na	na	na	na	0.057	-57.2%	0.91	na	na	na
$\sigma_{u_1u_2}$	0.072	0.137	190.3%	0.39	0.145	202.0%	0.33	0.002	3.1%	0.94	-0.015	-21.0%	0.96	na	na	na
$\sigma_{u_1u_3}$	0.072	0.060	83.0%	0.80	0.065	90.8%	0.80	0.007	10.4%	0.97	-0.003	-4.8%	0.97	na	na	na
$\sigma_{u_1u_4}$	-0.168	na	na	na	na	na	na	na	na	na	-0.054	32.0%	0.99	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.032	-268.9%	0.37	0.032	-268.8%	0.38	0.003	-27.3%	0.96	0.004	-30.3%	0.96	na	na	na
$\sigma_{u_2u_4}$	0.028	na	na	na	na	na	na	na	na	na	0.002	5.8%	1.00	na	na	na
$\sigma_{u_3u_4}$	-0.042	na	na	na	na	na	na	na	na	na	0.036	-84.6%	0.92	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations. na: not applicable. RelB: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. Cov: coverage for the 95% credible interval.

Table S2. Simulation study for change point generated data, under scenario 2 (moderate correlation between the random change point and the other individual-level random effects) using a sample size of 100 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.307	2.0%	0.53	0.289	1.9%	0.56	0.041	0.3%	0.97	0.026	0.2%	0.98	na	na	na
β_{20}	-0.4	0.098	-24.5%	0.26	0.082	-20.5%	0.56	0.007	-1.7%	0.96	-0.003	0.8%	0.96	na	na	na
β_{30}	0.6	-0.024	-4.0%	0.91	-0.028	-4.6%	0.90	-0.007	-1.1%	0.95	-0.002	-0.3%	0.96	na	na	na
ω_0	6.5	na	na	na	-0.079	-1.2%	0.70	-0.242	-3.7%	0.82	-0.138	-2.1%	0.89	-1.411	-21.7%	0.00
σ_y	1	0.027	2.7%	0.71	0.027	2.7%	0.74	0.002	0.2%	0.95	0.001	0.1%	0.98	0.043	4.3%	0.47
σ_{u_1}	1.2	0.250	20.8%	0.43	0.238	19.9%	0.51	0.024	2.0%	0.94	-0.007	-0.6%	0.98	na	na	na
σ_{u_2}	0.2	0.099	49.5%	0.17	0.101	50.3%	0.17	-0.040	-20.0%	0.90	-0.027	-13.5%	0.96	na	na	na
σ_{u_3}	0.3	0.036	12.0%	0.78	0.035	11.7%	0.77	0.022	7.4%	0.92	0.015	4.9%	0.95	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	-0.082	-5.9%	0.89	0.001	0.0%	0.96	na	na	na
$\rho_{u_1u_2}$	0.3	0.334	111.2%	0.11	0.323	107.6%	0.16	0.139	46.3%	0.83	-0.089	-29.6%	0.96	na	na	na
$\rho_{u_1u_3}$	0.2	0.161	80.4%	0.73	0.148	74.1%	0.77	0.051	25.6%	0.95	-0.009	-4.6%	0.97	na	na	na
$\rho_{u_1u_4}$	-0.3	na	na	na	na	na	na	na	na	na	-0.042	14.0%	0.98	na	na	na
$\rho_{u_2u_3}$	-0.2	0.483	-241.3%	0.07	0.465	-232.3%	0.09	0.140	-69.9%	0.90	0.060	-30.0%	0.95	na	na	na
$\rho_{u_2u_4}$	0.3	na	na	na	na	na	na	na	na	na	-0.187	-62.5%	0.99	na	na	na
$\rho_{u_3u_4}$	-0.3	na	na	na	na	na	na	na	na	na	0.112	-37.3%	0.91	na	na	na
$\sigma_{u_1u_2}$	0.072	0.213	295.5%	0.07	0.208	289.6%	0.11	0.028	38.3%	0.93	-0.017	-23.2%	0.95	na	na	na
$\sigma_{u_1u_3}$	0.072	0.104	144.9%	0.58	0.097	134.6%	0.65	0.026	36.5%	0.93	-0.001	-1.8%	0.96	na	na	na
$\sigma_{u_1u_4}$	-0.504	na	na	na	na	na	na	na	na	na	-0.054	10.8%	0.98	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.041	-337.6%	0.19	0.039	-322.3%	0.23	0.009	-74.3%	0.88	0.004	-35.5%	0.96	na	na	na
$\sigma_{u_2u_4}$	0.084	na	na	na	na	na	na	na	na	na	-0.029	-34.6%	0.96	na	na	na
$\sigma_{u_3u_4}$	-0.126	na	na	na	na	na	na	na	na	na	0.052	-41.3%	0.88	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations. na: not applicable. RelB: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. Cov: coverage for the 95% credible interval.

Table S3. Simulation study for change point generated data, under scenario 3 (strong correlation between the random change point and other individual-level random effects) using a sample size of 100 children for each simulated dataset.

Parameter	True values	Model 1			Model 2			Model 3a, uncorrelated			Model 3b, correlated			Model 4		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.324	2.2%	0.54	0.254	1.7%	0.70	0.069	0.5%	0.95	0.017	0.1%	0.98	na	na	na
β_{20}	-0.4	0.115	-28.8%	0.11	0.067	-16.6%	0.65	0.025	-6.2%	0.89	0.001	-0.3%	0.95	na	na	na
β_{30}	0.6	-0.013	-2.2%	0.96	-0.029	-4.9%	0.91	-0.010	-1.6%	0.96	0.000	0.0%	0.96	na	na	na
ω_0	6.5	na	na	na	-0.282	-4.3%	0.57	-0.313	-4.8%	0.70	-0.109	-1.7%	0.91	-1.522	-23.4%	0.00
σ_y	1	0.025	2.5%	0.76	0.024	2.4%	0.80	0.002	0.2%	0.98	-0.001	-0.1%	0.98	0.044	4.4%	0.47
σ_{u_1}	1.2	0.350	29.2%	0.20	0.290	24.1%	0.37	0.054	4.5%	0.93	-0.010	-0.8%	0.97	na	na	na
σ_{u_2}	0.2	0.079	39.5%	0.32	0.080	39.8%	0.36	-0.049	-24.7%	0.81	-0.037	-18.3%	0.94	na	na	na
σ_{u_3}	0.3	0.044	14.7%	0.68	0.043	14.3%	0.68	0.031	10.5%	0.79	0.011	3.8%	0.91	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	-0.251	-17.9%	0.67	-0.050	-3.5%	0.94	na	na	na
$\rho_{u_1u_2}$	0.3	0.505	168.4%	0.00	0.484	161.3%	0.02	0.380	126.8%	0.39	-0.077	-25.7%	0.97	na	na	na
$\rho_{u_1u_3}$	0.2	0.208	104.1%	0.51	0.183	91.3%	0.58	0.098	48.8%	0.87	-0.007	-3.6%	0.97	na	na	na
$\rho_{u_1u_4}$	-0.5	na	na	na	na	na	na	na	na	na	-0.066	13.2%	0.97	na	na	na
$\rho_{u_2u_3}$	-0.2	0.615	-307.3%	0.04	0.563	-281.7%	0.07	0.338	-169.0%	0.74	0.057	-28.3%	0.97	na	na	na
$\rho_{u_2u_4}$	0.5	na	na	na	na	na	na	na	na	na	-0.194	-38.8%	0.97	na	na	na
$\rho_{u_3u_4}$	-0.5	na	na	na	na	na	na	na	na	na	0.088	-17.7%	0.93	na	na	na
$\sigma_{u_1u_2}$	0.072	0.286	397.9%	0.00	0.266	368.8%	0.03	0.068	94.1%	0.77	-0.019	-26.7%	0.95	na	na	na
$\sigma_{u_1u_3}$	0.072	0.149	206.4%	0.29	0.126	174.9%	0.46	0.052	72.7%	0.83	-0.001	-1.0%	0.96	na	na	na
$\sigma_{u_1u_4}$	-0.84	na	na	na	na	na	na	na	na	na	-0.062	7.4%	1.00	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.052	-434.1%	0.06	0.047	-392.2%	0.12	0.019	-158.8%	0.62	0.004	-32.7%	0.95	na	na	na
$\sigma_{u_2u_4}$	0.14	na	na	na	na	na	na	na	na	na	-0.049	-34.9%	0.89	na	na	na
$\sigma_{u_3u_4}$	-0.21	na	na	na	na	na	na	na	na	na	0.042	-20.0%	0.87	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations. na: not applicable. RelB: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. Cov: coverage for the 95% credible interval.

Table S4. Simulation study for change point generated data, under scenario 1 (weak correlation between the random change point and other subject-specific random effects) using a sample size of 50 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.274	1.8%	0.75	0.296	2.0%	0.75	0.012	0.1%	0.95	0.018	0.1%	0.97	na	na	na
β_{20}	-0.4	0.069	-17.3%	0.78	0.079	-19.8%	0.74	-0.017	4.3%	0.94	-0.006	1.5%	0.97	na	na	na
β_{30}	0.6	-0.038	-6.3%	0.88	-0.029	-4.8%	0.95	-0.007	-1.1%	0.95	0.007	1.2%	0.98	na	na	na
ω_0	6.5	na	na	na	0.114	1.7%	0.75	-0.155	-2.4%	0.93	-0.090	-1.4%	0.97	-1.248	-19.2%	0.01
σ_y	1	0.033	3.3%	0.77	0.031	3.1%	0.78	0.004	0.4%	0.97	0.006	0.6%	0.94	0.053	5.3%	0.56
σ_{u_1}	1.2	0.172	14.3%	0.81	0.191	15.9%	0.79	0.022	1.8%	0.92	-0.023	-1.9%	0.95	na	na	na
σ_{u_2}	0.2	0.126	62.8%	0.23	0.124	62.2%	0.24	-0.013	-6.5%	0.95	-0.020	-10.2%	0.97	na	na	na
σ_{u_3}	0.3	0.039	13.0%	0.86	0.042	13.8%	0.85	0.022	7.2%	0.92	0.021	7.0%	0.93	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	0.044	3.1%	0.95	0.123	8.8%	0.94	na	na	na
$\rho_{u_1u_2}$	0.3	0.122	40.5%	0.83	0.137	45.5%	0.81	-0.066	-21.9%	0.99	-0.178	-59.3%	0.99	na	na	na
$\rho_{u_1u_3}$	0.2	0.086	43.2%	0.94	0.090	45.1%	0.94	0.008	4.2%	0.94	0.023	11.4%	0.98	na	na	na
$\rho_{u_1u_4}$	-0.1	na	na	na	na	na	na	na	na	na	-0.065	65.0%	0.99	na	na	na
$\rho_{u_2u_3}$	-0.2	0.336	-168.0%	0.58	0.329	-164.3%	0.63	0.000	-0.2%	1.00	0.147	-73.3%	0.99	na	na	na
$\rho_{u_2u_4}$	0.1	na	na	na	na	na	na	na	na	na	-0.124	-123.8%	1.00	na	na	na
$\rho_{u_3u_4}$	-0.1	na	na	na	na	na	na	na	na	na	0.090	-89.8%	0.95	na	na	na
$\sigma_{u_1u_2}$	0.072	0.135	187.8%	0.70	0.145	201.8%	0.66	0.007	9.8%	0.94	-0.028	-38.9%	0.95	na	na	na
$\sigma_{u_1u_3}$	0.072	0.059	81.3%	0.91	0.063	87.8%	0.89	0.006	7.7%	0.92	0.010	14.4%	0.97	na	na	na
$\sigma_{u_1u_4}$	-0.168	na	na	na	na	na	na	na	na	na	-0.112	66.5%	0.99	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.026	-217.6%	0.80	0.025	-211.6%	0.79	0.000	3.6%	1.00	0.008	-64.4%	0.95	na	na	na
$\sigma_{u_2u_4}$	0.028	na	na	na	na	na	na	na	na	na	-0.004	-13.6%	1.00	na	na	na
$\sigma_{u_3u_4}$	-0.042	na	na	na	na	na	na	na	na	na	0.060	-142.8%	0.96	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations: "na": not applicable. "RelB": relative bias, calculated as bias divided by the true parameter value and represented as a percentage. "Cov": coverage for the 95% credible interval.

Table S5. Simulation study for change point generated data, under scenario 2 (moderate correlation between the random change point and other subject-specific random effects) using a sample size of 50 children for each simulated dataset.

Parameter	**True values**		**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**	
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.273	1.8%	0.77	0.257	1.7%	0.78	0.002	0.0%	0.97	-0.010	-0.1%	0.96	na	na	na
β_{20}	-0.4	0.087	-21.7%	0.68	0.071	-17.7%	0.78	-0.008	1.9%	0.96	-0.022	5.4%	0.95	na	na	na
β_{30}	0.6	-0.021	-3.4%	0.93	-0.024	-3.9%	0.94	-0.004	-0.7%	0.98	0.002	0.4%	0.99	na	na	na
ω_0	6.5	na	na	na	-0.059	-0.9%	0.77	-0.251	-3.9%	0.93	-0.162	-2.5%	0.96	-1.366	-21.0%	0.00
σ_y	1	0.029	2.9%	0.84	0.028	2.8%	0.84	0.003	0.3%	0.95	0.003	0.3%	0.95	0.051	5.1%	0.63
σ_{u_1}	1.2	0.272	22.7%	0.62	0.263	21.9%	0.63	0.035	2.9%	0.91	0.014	1.2%	0.94	na	na	na
σ_{u_2}	0.2	0.115	57.5%	0.31	0.115	57.5%	0.32	-0.026	-13.2%	0.95	-0.009	-4.7%	0.97	na	na	na
σ_{u_3}	0.3	0.057	18.9%	0.71	0.056	18.7%	0.72	0.040	13.3%	0.82	0.033	11.1%	0.89	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	-0.046	-3.3%	0.87	0.076	5.4%	0.94	na	na	na
$\rho_{u_1u_2}$	0.3	0.296	98.8%	0.40	0.283	94.3%	0.49	0.083	27.7%	0.93	-0.104	-34.5%	1.00	na	na	na
$\rho_{u_1u_3}$	0.2	0.118	59.1%	0.89	0.107	53.6%	0.90	0.016	8.0%	0.97	-0.045	-22.6%	0.92	na	na	na
$\rho_{u_1u_4}$	-0.3	na	na	na	na	na	na	na	na	na	-0.011	3.7%	0.96	na	na	na
$\rho_{u_2u_3}$	-0.2	0.389	-194.3%	0.47	0.369	-184.7%	0.57	0.057	-28.4%	1.00	0.022	-10.8%	1.00	na	na	na
$\rho_{u_2u_4}$	0.3	na	na	na	na	na	na	na	na	na	-0.164	-54.8%	1.00	na	na	na
$\rho_{u_3u_4}$	-0.3	na	na	na	na	na	na	na	na	na	0.160	-53.4%	0.95	na	na	na
$\sigma_{u_1u_2}$	0.072	0.230	319.0%	0.28	0.225	312.2%	0.37	0.034	47.5%	0.97	-0.005	-6.8%	0.96	na	na	na
$\sigma_{u_1u_3}$	0.072	0.095	132.2%	0.84	0.088	122.3%	0.85	0.015	21.3%	0.96	-0.013	-18.3%	0.93	na	na	na
$\sigma_{u_1u_4}$	-0.504	na	na	na	na	na	na	na	na	na	-0.067	13.4%	0.95	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.033	-279.1%	0.68	0.031	-260.8%	0.77	0.003	-25.2%	0.99	0.001	-6.2%	1.00	na	na	na
$\sigma_{u_2u_4}$	0.084	na	na	na	na	na	na	na	na	na	-0.006	-7.3%	1.00	na	na	na
$\sigma_{u_3u_4}$	-0.126	na	na	na	na	na	na	na	na	na	0.078	-61.7%	0.94	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations: "na": not applicable. "RelB": relative bias, calculated as bias divided by the true parameter value and represented as a percentage. "Cov": coverage for the 95% credible interval.

Table S6. Simulation study for change point generated data, under scenario 3 (strong correlation between the random change point and other subject-specific random effects) using a sample size of 50 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.308	2.1%	0.78	0.240	1.6%	0.88	0.046	0.3%	0.97	-0.004	0.0%	0.98	na	na	na
β_{20}	-0.4	0.113	-28.3%	0.40	0.063	-15.8%	0.81	0.019	-4.8%	0.95	-0.006	1.5%	0.96	na	na	na
β_{30}	0.6	-0.009	-1.5%	0.96	-0.024	-4.0%	0.92	-0.004	-0.7%	0.96	0.004	0.6%	0.97	na	na	na
ω_0	6.5	na	na	na	-0.275	-4.2%	0.64	-0.318	-4.9%	0.82	-0.164	-2.5%	0.91	-1.516	-23.3%	0.00
σ_y	1	0.032	3.2%	0.82	0.031	3.1%	0.84	0.010	1.0%	0.99	0.007	0.7%	0.99	0.055	5.5%	0.60
σ_{u_1}	1.2	0.363	30.3%	0.45	0.306	25.5%	0.59	0.068	5.7%	0.93	0.010	0.8%	0.96	na	na	na
σ_{u_2}	0.2	0.084	41.8%	0.55	0.085	42.5%	0.57	-0.042	-20.8%	0.90	-0.039	-19.6%	0.91	na	na	na
σ_{u_3}	0.3	0.061	20.2%	0.75	0.059	19.6%	0.76	0.043	14.5%	0.87	0.026	8.5%	0.97	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	-0.215	-15.3%	0.84	-0.025	-1.8%	0.95	na	na	na
$\rho_{u_1u_2}$	0.3	0.494	164.5%	0.05	0.471	157.1%	0.11	0.290	96.7%	0.78	-0.074	-24.8%	0.97	na	na	na
$\rho_{u_1u_3}$	0.2	0.157	78.4%	0.90	0.133	66.6%	0.94	0.058	29.2%	0.99	-0.050	-25.0%	0.98	na	na	na
$\rho_{u_1u_4}$	-0.5	na	na	na	na	na	na	na	na	na	-0.047	9.4%	0.97	na	na	na
$\rho_{u_2u_3}$	-0.2	0.535	-267.6%	0.17	0.483	-241.3%	0.33	0.234	-117.2%	0.93	0.048	-23.8%	0.98	na	na	na
$\rho_{u_2u_4}$	0.5	na	na	na	na	na	na	na	na	na	-0.263	-52.5%	0.98	na	na	na
$\rho_{u_3u_4}$	-0.5	na	na	na	na	na	na	na	na	na	0.158	-31.6%	0.97	na	na	na
$\sigma_{u_1u_2}$	0.072	0.301	417.5%	0.11	0.282	391.2%	0.13	0.076	106.1%	0.90	-0.008	-11.3%	0.94	na	na	na
$\sigma_{u_1u_3}$	0.072	0.132	183.0%	0.72	0.111	154.0%	0.80	0.041	56.8%	0.98	-0.014	-19.3%	0.98	na	na	na
$\sigma_{u_1u_4}$	-0.84	na	na	na	na	na	na	na	na	na	-0.073	8.7%	0.96	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.047	-390.0%	0.33	0.041	-345.6%	0.49	0.015	-122.5%	0.86	0.003	-25.5%	0.97	na	na	na
$\sigma_{u_2u_4}$	0.14	na	na	na	na	na	na	na	na	na	-0.057	-40.5%	0.91	na	na	na
$\sigma_{u_3u_4}$	-0.21	na	na	na	na	na	na	na	na	na	0.066	-31.3%	0.94	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define “bias” as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations: “na”: not applicable. “RelB”: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. “Cov”: coverage for the 95% credible interval.

Table S7. Simulation study for change point generated data, under scenario 1 (weak correlation between the random change point and other subject-specific random effects) using a sample size of 150 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.234	1.6%	0.58	0.260	1.7%	0.54	-0.003	0.0%	0.96	-0.004	0.0%	0.96	na	na	na
β_{20}	-0.4	0.064	-16.1%	0.45	0.081	-20.2%	0.42	-0.003	0.7%	0.95	-0.006	1.5%	0.95	na	na	na
β_{30}	0.6	-0.034	-5.6%	0.80	-0.025	-4.2%	0.85	0.000	0.0%	0.96	0.002	0.3%	0.94	na	na	na
ω_0	6.5	na	na	na	0.122	1.9%	0.71	-0.067	-1.0%	0.93	-0.020	-0.3%	0.97	-1.242	-19.1%	0.00
σ_y	1	0.034	3.4%	0.45	0.033	3.3%	0.51	0.003	0.3%	0.92	0.003	0.3%	0.95	0.049	4.9%	0.20
σ_{u_1}	1.2	0.145	12.1%	0.59	0.164	13.7%	0.56	0.016	1.3%	0.96	0.009	0.7%	0.96	na	na	na
σ_{u_2}	0.2	0.107	53.6%	0.05	0.105	52.6%	0.05	-0.013	-6.3%	0.93	-0.006	-2.8%	0.94	na	na	na
σ_{u_3}	0.3	0.021	7.0%	0.85	0.023	7.7%	0.83	0.010	3.3%	0.95	0.008	2.5%	0.97	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	-0.020	-1.4%	0.93	0.015	1.1%	0.91	na	na	na
$\rho_{u_1u_2}$	0.3	0.153	50.8%	0.64	0.169	56.2%	0.60	0.007	2.4%	0.95	-0.063	-21.0%	0.97	na	na	na
$\rho_{u_1u_3}$	0.2	0.085	42.5%	0.83	0.093	46.7%	0.79	-0.002	-1.1%	0.94	-0.025	-12.7%	0.94	na	na	na
$\rho_{u_1u_4}$	-0.1	na	na	na	na	na	na	na	na	na	-0.019	18.8%	0.94	na	na	na
$\rho_{u_2u_3}$	-0.2	0.406	-202.9%	0.03	0.410	-204.9%	0.04	0.032	-16.2%	0.95	0.021	-10.3%	0.95	na	na	na
$\rho_{u_2u_4}$	0.1	na	na	na	na	na	na	na	na	na	-0.082	-82.4%	0.94	na	na	na
$\rho_{u_3u_4}$	-0.1	na	na	na	na	na	na	na	na	na	0.022	-21.8%	0.94	na	na	na
$\sigma_{u_1u_2}$	0.072	0.122	169.0%	0.29	0.131	181.3%	0.26	0.006	7.7%	0.94	-0.008	-10.5%	0.96	na	na	na
$\sigma_{u_1u_3}$	0.072	0.050	69.8%	0.75	0.057	79.1%	0.70	0.001	1.5%	0.96	-0.008	-11.5%	0.94	na	na	na
$\sigma_{u_1u_4}$	-0.168	na	na	na	na	na	na	na	na	na	-0.015	8.9%	0.95	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.032	-267.9%	0.20	0.033	-271.0%	0.15	0.002	-19.7%	0.97	0.002	-13.8%	0.96	na	na	na
$\sigma_{u_2u_4}$	0.028	na	na	na	na	na	na	na	na	na	0.006	21.0%	0.92	na	na	na
$\sigma_{u_3u_4}$	-0.042	na	na	na	na	na	na	na	na	na	0.017	-40.8%	0.93	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define “bias” as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations: “na”: not applicable. “RelB”: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. “Cov”: coverage for the 95% credible interval.

Table S8. Simulation study for change point generated data, under scenario 2 (moderate correlation between the random change point and other subject-specific random effects) using a sample size of 150 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.263	1.8%	0.46	0.246	1.6%	0.57	0.010	0.1%	0.97	-0.006	0.0%	0.96	na	na	na
β_{20}	-0.4	0.087	-21.7%	0.19	0.073	-18.2%	0.50	0.005	-1.2%	0.94	-0.004	1.0%	0.93	na	na	na
β_{30}	0.6	-0.024	-4.1%	0.87	-0.028	-4.7%	0.85	-0.005	-0.8%	0.96	0.000	0.1%	0.93	na	na	na
ω_0	6.5	na	na	na	-0.073	-1.1%	0.73	-0.187	-2.9%	0.85	-0.044	-0.7%	0.95	-1.362	-21.0%	0.00
σ_y	1	0.031	3.1%	0.56	0.031	3.1%	0.59	0.004	0.4%	0.94	0.003	0.3%	0.97	0.047	4.7%	0.25
σ_{u_1}	1.2	0.223	18.5%	0.32	0.211	17.5%	0.40	0.021	1.8%	0.89	-0.005	-0.4%	0.93	na	na	na
σ_{u_2}	0.2	0.094	47.0%	0.09	0.095	47.3%	0.09	-0.034	-17.0%	0.92	-0.016	-7.9%	0.93	na	na	na
σ_{u_3}	0.3	0.033	11.2%	0.64	0.033	11.0%	0.68	0.020	6.6%	0.88	0.009	2.8%	0.95	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	-0.092	-6.6%	0.90	-0.008	-0.5%	0.91	na	na	na
$\rho_{u_1u_2}$	0.3	0.322	107.3%	0.04	0.312	104.2%	0.05	0.159	52.9%	0.80	-0.064	-21.3%	0.96	na	na	na
$\rho_{u_1u_3}$	0.2	0.157	78.3%	0.57	0.146	72.8%	0.64	0.043	21.6%	0.92	-0.017	-8.6%	0.93	na	na	na
$\rho_{u_1u_4}$	-0.3	na	na	na	na	na	na	na	na	na	-0.022	7.3%	0.93	na	na	na
$\rho_{u_2u_3}$	-0.2	0.500	-250.2%	0.01	0.485	-242.3%	0.01	0.139	-69.6%	0.89	0.022	-11.1%	0.97	na	na	na
$\rho_{u_2u_4}$	0.3	na	na	na	na	na	na	na	na	na	-0.142	-47.2%	0.93	na	na	na
$\rho_{u_3u_4}$	-0.3	na	na	na	na	na	na	na	na	na	0.050	-16.8%	0.94	na	na	na
$\sigma_{u_1u_2}$	0.072	0.195	270.8%	0.02	0.190	264.1%	0.03	0.029	39.9%	0.88	-0.011	-15.8%	0.95	na	na	na
$\sigma_{u_1u_3}$	0.072	0.098	135.7%	0.42	0.091	126.5%	0.47	0.022	30.3%	0.91	-0.006	-8.2%	0.94	na	na	na
$\sigma_{u_1u_4}$	-0.504	na	na	na	na	na	na	na	na	na	-0.017	3.3%	0.92	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.041	-345.1%	0.04	0.040	-332.4%	0.07	0.009	-71.8%	0.82	0.002	-15.3%	0.96	na	na	na
$\sigma_{u_2u_4}$	0.084	na	na	na	na	na	na	na	na	na	-0.015	-18.0%	0.92	na	na	na
$\sigma_{u_3u_4}$	-0.126	na	na	na	na	na	na	na	na	na	0.025	-20.1%	0.90	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations: "na": not applicable. "RelB": relative bias, calculated as bias divided by the true parameter value and represented as a percentage. "Cov": coverage for the 95% credible interval.

Table S9. Simulation study for change point generated data, under scenario 3 (strong correlation between the random change point and other subject-specific random effects) using a sample size of 150 children for each simulated dataset.

Parameter	True values	Model 1			Model 2			Model 3a, uncorrelated			Model 3b, correlated			Model 4		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
β_{10}	15	0.288	1.9%	0.43	0.231	1.5%	0.61	0.050	0.3%	0.94	-0.004	0.0%	0.96	na	na	na
β_{20}	-0.4	0.109	-27.4%	0.05	0.071	-17.7%	0.54	0.030	-7.6%	0.77	0.003	-0.8%	0.92	na	na	na
β_{30}	0.6	-0.017	-2.8%	0.90	-0.031	-5.1%	0.85	-0.010	-1.7%	0.96	-0.002	-0.4%	0.98	na	na	na
ω_0	6.5	na	na	na	-0.235	-3.6%	0.52	-0.265	-4.1%	0.70	-0.057	-0.9%	0.97	-1.492	-23.0%	0.00
σ_y	1	0.029	2.9%	0.64	0.028	2.8%	0.65	0.006	0.6%	0.94	0.002	0.2%	0.94	0.045	4.5%	0.31
σ_{u_1}	1.2	0.310	25.8%	0.14	0.260	21.7%	0.30	0.053	4.5%	0.87	-0.013	-1.0%	0.97	na	na	na
σ_{u_2}	0.2	0.073	36.6%	0.25	0.074	36.8%	0.30	-0.045	-22.7%	0.72	-0.025	-12.3%	0.89	na	na	na
σ_{u_3}	0.3	0.040	13.3%	0.53	0.039	12.8%	0.57	0.026	8.8%	0.80	0.005	1.8%	0.94	na	na	na
σ_{u_4}	1.4	na	na	na	na	na	na	-0.258	-18.5%	0.62	-0.050	-3.6%	0.95	na	na	na
$\rho_{u_1u_2}$	0.3	0.514	171.4%	0.00	0.496	165.5%	0.00	0.451	150.2%	0.15	-0.035	-11.6%	0.97	na	na	na
$\rho_{u_1u_3}$	0.2	0.208	104.1%	0.42	0.187	93.7%	0.47	0.095	47.3%	0.80	-0.018	-9.1%	0.95	na	na	na
$\rho_{u_1u_4}$	-0.5	na	na	na	na	na	na	na	na	na	-0.049	9.8%	0.92	na	na	na
$\rho_{u_2u_3}$	-0.2	0.610	-305.0%	0.00	0.572	-286.1%	0.01	0.338	-169.2%	0.60	0.022	-11.1%	0.97	na	na	na
$\rho_{u_2u_4}$	0.5	na	na	na	na	na	na	na	na	na	-0.151	-30.2%	0.96	na	na	na
$\rho_{u_3u_4}$	-0.5	na	na	na	na	na	na	na	na	na	0.069	-13.8%	0.95	na	na	na
$\sigma_{u_1u_2}$	0.072	0.270	375.4%	0.00	0.253	351.4%	0.00	0.079	110.4%	0.59	-0.007	-9.6%	0.93	na	na	na
$\sigma_{u_1u_3}$	0.072	0.139	192.8%	0.17	0.121	167.9%	0.26	0.048	67.3%	0.76	-0.007	-9.3%	0.96	na	na	na
$\sigma_{u_1u_4}$	-0.84	na	na	na	na	na	na	na	na	na	-0.031	3.7%	0.95	na	na	na
$\sigma_{u_2u_3}$	-0.012	0.050	-415.6%	0.01	0.046	-385.5%	0.02	0.019	-156.6%	0.53	0.002	-20.3%	0.94	na	na	na
$\sigma_{u_2u_4}$	0.14	na	na	na	na	na	na	na	na	na	-0.040	-28.3%	0.94	na	na	na
$\sigma_{u_3u_4}$	-0.21	na	na	na	na	na	na	na	na	na	0.035	-16.7%	0.90	na	na	na

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations: "na": not applicable. "RelB": relative bias, calculated as bias divided by the true parameter value and represented as a percentage. "Cov": coverage for the 95% credible interval.

Table S10. Simulation study for fractional polynomial generated data, using a sample size of 100 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
α_{10}	20	na	na	na	na	na	na	na	na	na	na	na	na	-0.029	-0.1%	0.92
α_{20}	-2.1	na	na	na	na	na	na	na	na	na	na	na	na	0.010	-0.5%	0.93
α_{30}	0.73	na	na	na	na	na	na	na	na	na	na	na	na	-0.004	-0.5%	0.92
ω_0	6.532	na	na	na	-0.081	-1.2%	0.75	-0.124	-1.9%	0.80	-0.138	-2.1%	0.92	0.005	0.1%	0.94
σ_y	1	0.029	2.9%	0.72	0.028	2.8%	0.72	0.018	1.8%	0.84	0.011	1.1%	0.93	0.007	0.7%	0.94
σ_{u_1}	2	na	na	na	na	na	na	na	na	na	na	na	na	-0.121	-6.1%	0.92
σ_{u_2}	1	na	na	na	na	na	na	na	na	na	na	na	na	-0.078	-7.8%	0.86
σ_{u_3}	0.3	na	na	na	na	na	na	na	na	na	na	na	na	-0.025	-8.2%	0.86
$\rho_{u_1u_2}$	-0.83	na	na	na	na	na	na	na	na	na	na	na	na	0.055	-6.6%	0.93
$\rho_{u_1u_3}$	0.77	na	na	na	na	na	na	na	na	na	na	na	na	-0.077	-10.0%	0.92
$\rho_{u_2u_3}$	-0.96	na	na	na	na	na	na	na	na	na	na	na	na	0.017	-1.8%	0.93
$\sigma_{u_1u_2}$	-1.66	na	na	na	na	na	na	na	na	na	na	na	na	0.222	-13.4%	0.90
$\sigma_{u_1u_3}$	0.462	na	na	na	na	na	na	na	na	na	na	na	na	-0.069	-15.0%	0.91
$\sigma_{u_2u_3}$	-0.288	na	na	na	na	na	na	na	na	na	na	na	na	0.036	-12.6%	0.86

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations. na: not applicable. RelB: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. Cov: coverage for the 95% credible interval.

Table S11. Simulation study for fractional polynomial generated data, using a sample size of 50 children for each simulated dataset.

Parameter	**True values**	**Model 1**			**Model 2**			**Model 3a, uncorrelated**			**Model 3b, correlated**			**Model 4**		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
α_{10}	20	na	na	na	na	na	na	na	na	na	na	na	na	-0.082	-0.4%	0.93
α_{20}	-2.1	na	na	na	na	na	na	na	na	na	na	na	na	0.032	-1.5%	0.93
α_{30}	0.73	na	na	na	na	na	na	na	na	na	na	na	na	-0.009	-1.3%	0.93
ω_0	6.532	na	na	na	0.000	0.0%	0.78	-0.118	-1.8%	0.88	-0.279	-4.3%	0.86	-0.051	-0.8%	0.93
σ_y	1	0.029	2.9%	0.83	0.027	2.7%	0.84	0.018	1.8%	0.93	0.013	1.3%	0.95	0.012	1.2%	0.95
σ_{u_1}	2	na	na	na	na	na	na	na	na	na	na	na	na	-0.265	-13.3%	0.91
σ_{u_2}	1	na	na	na	na	na	na	na	na	na	na	na	na	-0.108	-10.8%	0.94
σ_{u_3}	0.3	na	na	na	na	na	na	na	na	na	na	na	na	-0.035	-11.6%	0.93
$\rho_{u_1u_2}$	-0.83	na	na	na	na	na	na	na	na	na	na	na	na	0.146	-17.6%	0.87
$\rho_{u_1u_3}$	0.77	na	na	na	na	na	na	na	na	na	na	na	na	-0.192	-25.0%	0.87
$\rho_{u_2u_3}$	-0.96	na	na	na	na	na	na	na	na	na	na	na	na	0.040	-4.2%	0.91
$\sigma_{u_1u_2}$	-1.66	na	na	na	na	na	na	na	na	na	na	na	na	0.376	-22.6%	0.88
$\sigma_{u_1u_3}$	0.462	na	na	na	na	na	na	na	na	na	na	na	na	-0.117	-25.4%	0.86
$\sigma_{u_2u_3}$	-0.288	na	na	na	na	na	na	na	na	na	na	na	na	0.045	-15.5%	0.93

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations. na: not applicable. RelB: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. Cov: coverage for the 95% credible interval.

Table S12. Simulation study for fractional polynomial generated data, using a sample size of 150 children for each simulated dataset.

Parameter	True values	Model 1			Model 2			Model 3a, uncorrelated			Model 3b, correlated			Model 4		
		Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov	Bias	RelB	Cov
α_{10}	20	na	na	na	na	na	na	na	na	na	na	na	na	-0.004	0.0%	0.92
α_{20}	-2.1	na	na	na	na	na	na	na	na	na	na	na	na	0.005	-0.2%	0.93
α_{30}	0.73	na	na	na	na	na	na	na	na	na	na	na	na	-0.002	-0.3%	0.93
ω_0	6.532	na	na	na	-0.098	-1.5%	0.78	-0.122	-1.9%	0.80	-0.141	-2.2%	0.87	0.006	0.1%	0.98
σ_y	1	0.025	2.5%	0.68	0.024	2.4%	0.73	0.016	1.6%	0.91	0.008	0.8%	0.94	0.003	0.3%	0.97
σ_{u_1}	2	na	na	na	na	na	na	na	na	na	na	na	na	-0.130	-6.5%	0.90
σ_{u_2}	1	na	na	na	na	na	na	na	na	na	na	na	na	-0.068	-6.8%	0.88
σ_{u_3}	0.3	na	na	na	na	na	na	na	na	na	na	na	na	-0.023	-7.6%	0.89
$\rho_{u_1u_2}$	-0.83	na	na	na	na	na	na	na	na	na	na	na	na	0.048	-5.8%	0.89
$\rho_{u_1u_3}$	0.77	na	na	na	na	na	na	na	na	na	na	na	na	-0.069	-8.9%	0.87
$\rho_{u_2u_3}$	-0.96	na	na	na	na	na	na	na	na	na	na	na	na	0.012	-1.3%	0.88
$\sigma_{u_1u_2}$	-1.66	na	na	na	na	na	na	na	na	na	na	na	na	0.235	-14.1%	0.91
$\sigma_{u_1u_3}$	0.462	na	na	na	na	na	na	na	na	na	na	na	na	-0.076	-16.4%	0.89
$\sigma_{u_2u_3}$	-0.288	na	na	na	na	na	na	na	na	na	na	na	na	0.035	-12.1%	0.89

Notes. Model 1: fixed change point, assumed known *a priori*. Model 2: fixed change point, estimated from the data. Model 3a, uncorrelated: random change point assumed to be independent of the other individual-level random effects. Model 3b, correlated: random change point allowed to be correlated with the other individual-level random effects. Model 4: fractional polynomial model. Results reported are the average (over 100 simulated datasets) absolute bias, relative bias and coverage for the 95% credible intervals. We define "bias" as the difference between the posterior mean for the parameter and the true parameter value. Abbreviations. na: not applicable. RelB: relative bias, calculated as bias divided by the true parameter value and represented as a percentage. Cov: coverage for the 95% credible interval.