Appendix

Establishment of spatial coordinates

The spatial coordinates were established as X (anterior-posterior), Y (medial-lateral), and Z (proximal-distal) (Figs. E-1 and E-2). The origin of coordinates O (0, 0, 0) was defined as the midpoint of the anteroposterior (AP) axis of the proximal part of the tibia for anteroposterior axes 1 and 2, and it was defined as the midpoint of Line P-CEA (a line paralleling the clinical epicondylar axis and traversing the longest distance on the proximal part of the tibia) in axes 3 to 5. The proximal end of the extramedullary guide was placed on the line of the extended anteroposterior axis of the proximal part of the tibia. The distance D was defined as the distance between the origin of coordinates and the proximal end of the extramedullary guide. The proximal end of the extramedullary guide was defined as A (D, 0, 0). The distance H was defined as the distance between the proximal part of the tibia and the ankle joint. The center of the ankle joint was defined as O' (0, 0, -H). Point B is the ideal position for the distal end of the extramedullary guide. This point is determined so that $\overline{AB}$ is parallel to $\overline{OO'}$ on X-Z plane (Y = 0 plane).

The predicted postoperative tibial coronal alignment was calculated with the distal end of the extramedullary guide placed in front of the center of the ankle joint (on the line of the extended anteroposterior axis of the ankle joint: the line $\overline{O'B'}$). The proximal end of the extramedullary guide was fixed, and the distance between the distal end of the extramedullary guide and the center of the ankle joint (O') was not changed (Fig. E-2). The distal end of the extramedullary guide was rotated from point B to B' by the axis of rotation between the proximal end of the extramedullary guide and the center of the ankle joint (Fig. E-2). This rotational angle was defined as the converted angle ω' and was expressed as the angle ω using the law of cosines. B (D, 0, -H) and B' were rotated by angle ω' as the basis for axis $\overline{AO'}$ .The foot of a perpendicular to axis $\overline{AO'}$ from B (D, 0, -H) was defined as C. From the law of similitude between triangle O'AB and BAC, $\overline{BC} = \dfrac{DH}{\sqrt{D^2+H^2}} = \overline{B'C}$.

Using the law of cosines for triangle O'BB',

$$\overline{(BB')}^2 = \overline{(O'B)}^2 + \overline{(O'B')}^2 - 2\overline{(O'B)}\,\overline{(O'B')}\cos\omega = 2D^2(1-\cos\omega)$$

Using the law of cosines for triangle CBB',

$$\cos\omega' = \frac{\overline{(BC)}^2 + \overline{(B'C)}^2 - \overline{(BB')}^2}{2(\overline{BC})(\overline{B'C})} = \frac{H^2 - (1-\cos\omega)(D^2+H^2)}{H^2}$$

$$\omega' = \mathrm{Arc}\cos\left[\frac{H^2 - (1-\cos\omega)(D^2+H^2)}{H^2}\right]$$

In this situation, the normal vector of a plane with posterior slope γ is Vector P(–sin γ, 0, cos γ). The axis of rotation between the proximal end of the extramedullary guide and the center of the ankle joint is (D, 0, H), and the unit vector U is

$$\left(\frac{D}{\sqrt{D^2+H^2}}, 0, \frac{H}{\sqrt{D^2+H^2}}\right) = (\alpha, 0, \beta).$$

By rotating angle ω' around the unit vector U, the normal vector of the plane with Vector P can be converted to

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} -\sin\gamma\{\cos\omega' + \alpha^2(1-\cos\omega')\} + \cos\gamma\{\alpha\beta(1-\cos\omega')\} \\ -\sin\gamma(-\beta\sin\omega') + \cos\gamma(\alpha\sin\omega') \\ -\sin\gamma\{\alpha\beta(1-\cos\omega')\} + \cos\gamma\{\cos\omega' + \beta^2(1-\cos\omega')\} \end{bmatrix}$$

Therefore, (Ax + By + Cz = 0) was defined as an equation representing the axis of rotation between the center of the proximal part of the tibia and the ankle joint.

Figure E-3 shows two cutting lines of the proximal part of the tibia: one was an ideal cutting line perpendicular to the tibial mechanical axis and passing through the center of the proximal part of the tibia, and the second was a cutting line affected by the rotational angle. The angle θ was calculated using the following equation:

$$\tan\theta = \frac{|Zq - Zr|}{|Yq - Yr|} = \frac{Zq - Zr}{Yr - Yq} = \frac{B}{C}\ (BYr + CZr = BYq + CZq = 0)$$

$$\theta = \mathrm{Arc}\tan\frac{-\sin\gamma(-\beta\sin\omega') + \cos\gamma(\alpha\sin\omega')}{-\sin\gamma\{\alpha\beta(1-\cos\omega')\} + \cos\gamma\{\cos\omega' + \beta^2(1-\cos\omega')\}}$$

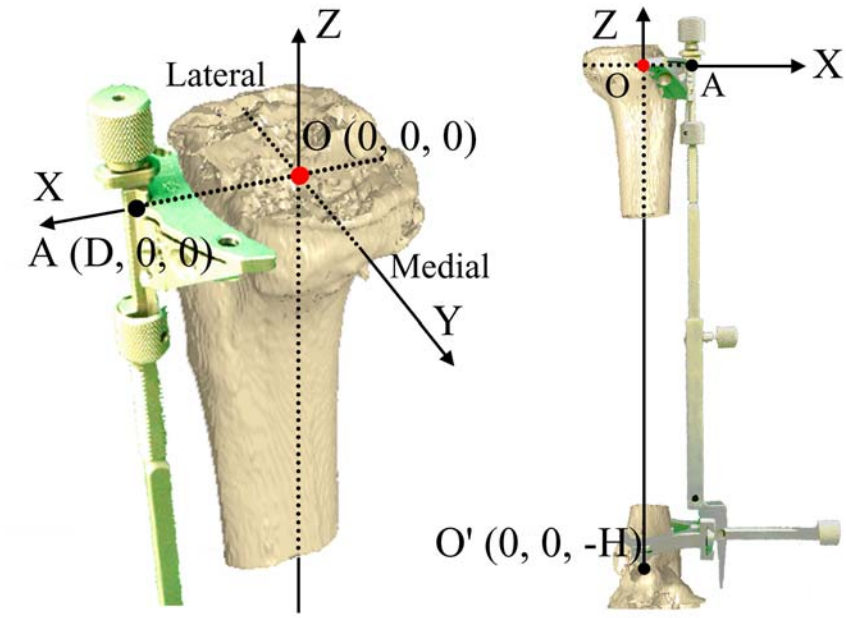

Fig. E-1

Extramedullary guide and the tibia. The center of the anteroposterior axis of the proximal part of the tibia on the bone surface was defined as O (0, 0, 0). The distance D was defined as the distance between the center of the anteroposterior axis of the proximal part of the tibia and the proximal end of the extramedullary guide. Thus, the proximal end of the extramedullary guide was established as A (D, 0, 0). The distance H was defined as the distance between the proximal part of the tibia and the ankle joint. The center of the ankle joint was thus established as O' (0, 0, –H).

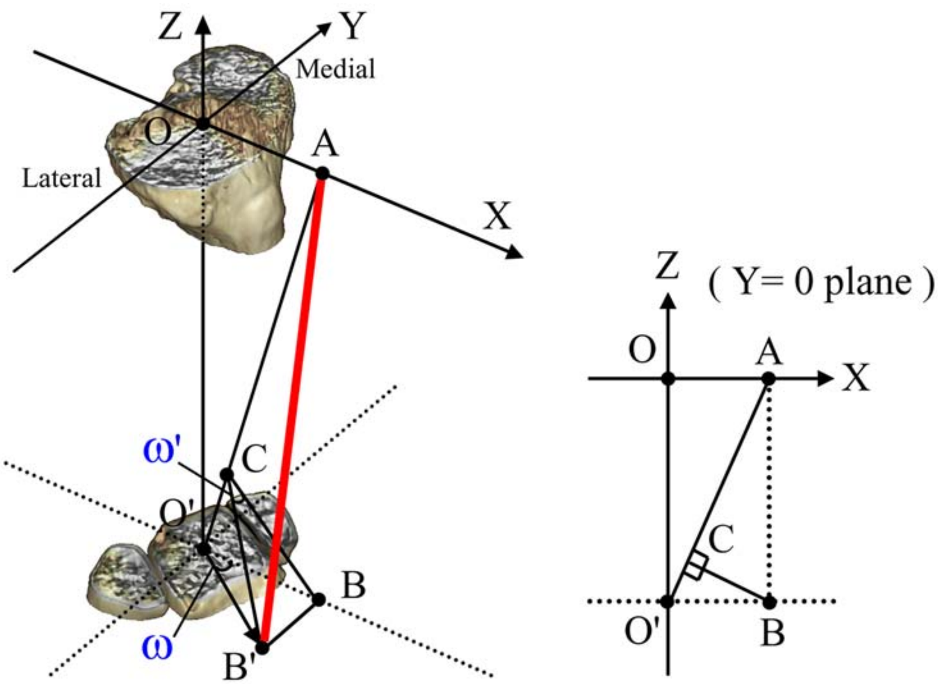

Fig. E-2

The relationship between the angle ω and the angle ω'. The distal end of the extramedullary guide was placed in front of the center of the ankle joint (on the line of the extended anteroposterior axis of the ankle joint [B']) under conditions with the proximal end of the extramedullary guide fixed (the red line).

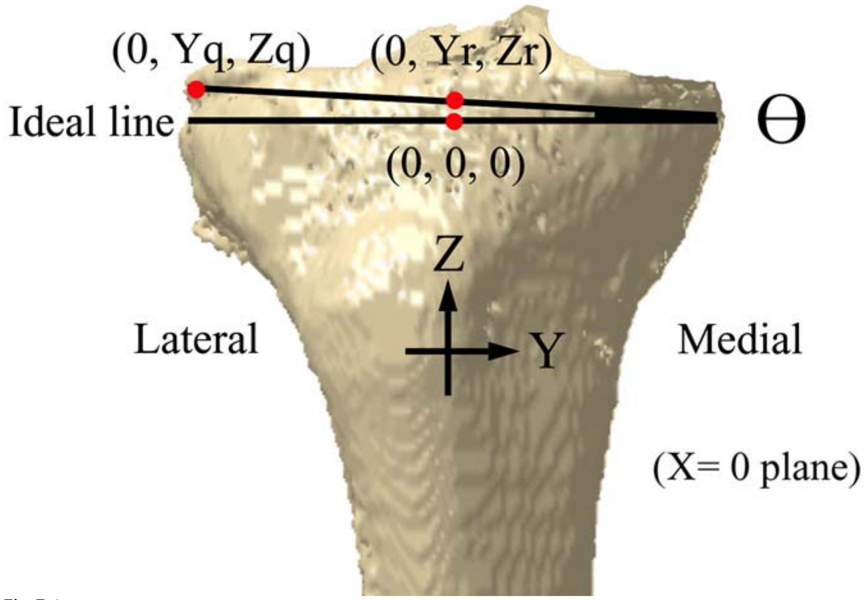

Fig. E-3

Calculation of angle θ (X = 0 plane). Plus means cutting the tibia in varus. A positive value indicates the tibia was cut in varus alignment.